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Hydromagnetic Effects on Fluid Velocity Components and Internal Flow Separation in an Exponentially Diverging Channel having Permeable Walls

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Abstract**:** *In this paper, we investigated the hydromagnetic steady flow of a viscous conducting fluid in a two-dimensional symmetrical slowly varying exponentially diverging channel having permeable walls. For this investigation the combined effects of an externally applied homogeneous magnetic field and permeable parameter on the development of velocity profiles and internal flow separation in the diverging channel are observed. The solution for the flow governing non-linear differential equation is found using perturbation method together with Pade´ approximation technique. The investigation results reveal that internal flow separation development at moderately large Reynolds number is suppressed by an increase in magnetic field intensity and decrease in permeable parameter. Furthermore, the behavior of velocity profiles under the effect of magnetic field and permeable parameter are discussed.*

Keywords: *Diverging channel, magnetic field, permeable parameter, internal flow separation, Pade´ approximants.*

Introduction

The study of electrically conducting viscous fluid flowing through diverging channels under the influence of an external magnetic field is not only fascinating theoretically, but also finds applications in mathematical modeling of several industrial and biological systems such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, the boundary layer control in the field of aerodynamics, blood flow problems, etc. Several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of MHD. A survey of MHD studies in the technological fields can be found in Moreau (1990). An investigation of MHD steady flow in a channel with slip at the permeable boundaries was carried out by Makinde and Osalusi (2006).

The theory of flow of convergent-divergent channels has many applications in aerospace, chemical, civil, environmental, mechanical and bio-mechanical engineering and also in understanding the flow of rivers and canals. The problem of laminar flow in channels of slowly varying width permeable boundaries was investigated in Makinde (1995). A numerical investigation of the study of hydromagnetic flows in a slowly varying exponentially diverging channel under the effect of an externally applied homogeneous magnetic field was conducted by Makinde and Mhone (2006), using perturbation method and Pade´ approximation technique, Baker (1975). Furthermore, internal flow separation due to hydromagnetic effects in a linearly diverging channel having permeable walls was found by Mallik *et al* (2011).

It is well known that, the flow separates at for values of Reynolds number above a rather moderate critical value if the cross-sectional area of a channel increases gradually with axial distance downstream. However the separated flow is not unique. Borgas and Pedley (1997) and Makinde (1997, 1999) showed analytically that this non-uniqueness occurs at large Reynolds number in channels that are sufficient slowly-varying for the flow to be governed by the boundary layer equations, in which there is neither a transverse pressure gradient nor longitudinal viscous diffusion.

In the present paper, the steady hydromagnetic flows in a two-dimensional symmetrical exponentially diverging channel having permeable walls under the influence of an externally applied homogeneous magnetic field have been investigated. The objective of the study is to analyze the behavior of fluid velocity profiles and to determine numerically the effects of the externally applied homogeneous magnetic field and permeable parameter on the development of internal flow separation as the flow Reynolds number increases using perturbation method together with Pade´ approximation technique.

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Mathematical Formulation

Consider the fluid flow where the fluid has small electrical conductivity and the electromagnetic force produced is very small under the effect of an externally applied homogeneous magnetic field. Let the fluid is flowing through a slowly varying exponentially diverging symmetrical channel having permeable walls as shown in Figure 1. Let u and v be the velocity components in the directions of x and y increasing respectively and $b(x)$ defines the wall diverging geometrically. Then, the governing equations for the two-dimensional steady flow, in terms of the vorticity (ω) and stream-function (ψ) can be written as

$$\frac{\partial(\omega,\psi)}{\partial(x,y)} = \nu\nabla^2\omega + \frac{\sigma_e {B_0}^2}{\rho}\frac{\partial^2\psi}{\partial y^2}, \qquad \omega = -\nabla^2\psi, \tag{2.1}$$

with the appropriate boundary conditions

$$\psi = 0, \quad \frac{\partial^2\psi}{\partial y^2} = 0 \quad \text{on} \quad y = 0, \tag{2.2}$$

and $$\psi = Q, \quad \frac{\partial\psi}{\partial y} = k\frac{db}{dx} \quad \text{on} \quad y = b(x) \tag{2.3}$$

where $Q = \int_0^{b(x)} u\,dy$ is the fluid flux rate across any section of the channel, k is the permeable parameter, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $B_0 = (\mu_e H_0)$ the electromagnetic induction, μ_e the magnetic permeability, H_0 the intensity of magnetic field, σ_e the conductivity of the fluid, ρ the fluid density and ν is the kinematic viscosity coefficient.

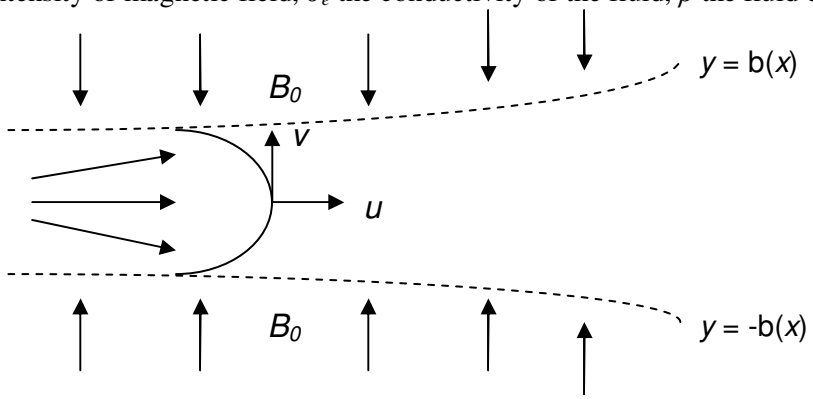

Figure 1: Geometry of the problem.

The axial and normal fluid velocity components in terms of the stream function can be written in the following manner

$$u = \frac{\partial\psi}{\partial y} \text{ and } \mathrm{v} = -\frac{\partial\psi}{\partial x}.$$

The inner surface of the wall is given by $y = \pm b(x/L)$. Let $b(x) = S(\varepsilon x/a)$ where S is the function of x, a is the characteristic half- width of the channel, ε is a small dimensionless parameter that specifies the slow variation in the cross-section of the channel defined as $0 < \varepsilon = a/L << 1$, where L is the channel characteristic length. In the limit $\varepsilon \to 0$, the channel is of constant width. The introduced dimensionless variables are

$$\bar{\omega} = \frac{a^2\omega}{Q},\ \bar{x} = \frac{\varepsilon x}{a},\ \bar{y} = \frac{y}{a},\ \bar{\psi} = \frac{\psi}{Q} \text{ and } H^2 = \frac{La\sigma_e {B_0}^2}{\rho Q}. \tag{2.4}$$

Hence the reduced dimensionless governing equations with the boundary conditions, (neglecting the bars for clarity) can be written as

$$\frac{\partial^2 \omega}{\partial y^2} = \mathrm{Re}\left[\frac{\partial(\omega,\psi)}{\partial(x,y)} - H^2 \frac{\partial^2 \psi}{\partial y^2}\right], \quad \omega = -\frac{\partial^2 \psi}{\partial y^2}, \tag{2.5}$$

$$\psi = 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \text{on} \quad y = 0, \tag{2.6}$$

$$\psi = 1, \quad \frac{\partial \psi}{\partial y} = k \frac{db}{dx} \quad \text{on} \quad y = S. \tag{2.7}$$

where the flow is considered in the boundary layer approximation or for channel with a small aspect ratio ε, $\mathrm{Re} = \varepsilon Q / \nu$ is the effective flow Reynolds number and H is the magnetic field intensity parameter or Hartmann number. For the geometry of the channel under consideration, S is defined as $S = e^x$.

Perturbation Expansion

The equations (2.5)-(2.7) are non-linear in nature and therefore it is not possible to find their solutions exactly. However, the solutions can be found in the form of power series in Re i.e.,

$$\psi = \sum_{i=0}^{\infty} \mathrm{Re}^i \psi_i, \qquad \omega = \sum_{i=0}^{\infty} \mathrm{Re}^i \omega_i. \tag{3.1}$$

Now substitute the expressions in (3.1) into (2.5)-(2.7) and collect the coefficients of like powers of Re. The resulting equations are:

Zeroth Order :

$$\frac{\partial^2 \omega_0}{\partial y^2} = 0, \quad \omega_0 = -\frac{\partial^2 \psi_0}{\partial y^2}, \tag{3.2}$$

$$\psi_0 = 0, \quad \frac{\partial^2 \psi_0}{\partial y^2} = 0, \quad \text{on} \quad y = 0, \tag{3.3}$$

$$\psi_0 = 1, \quad \frac{\partial \psi_0}{\partial y} = k \frac{db}{dx}, \quad \text{on} \quad y = S. \tag{3.4}$$

Higher Order ($n \geq 1$) :

$$\frac{\partial^2 \omega_n}{\partial y^2} = \sum_{i=0}^{n-1} \frac{\partial\left(\omega_i, \psi_{n-i-1}\right)}{\partial(x,y)} - H^2 \frac{\partial^2 \psi_{n-1}}{\partial y^2}, \quad \omega_n = -\frac{\partial^2 \psi_n}{\partial y^2}, \tag{3.5}$$

$$\psi_n = 0, \quad \frac{\partial^2 \psi_n}{\partial y^2} = 0, \quad \text{on} \quad y = 0, \tag{3.6}$$

$$\frac{\partial \psi_n}{\partial y} = 0, \quad \psi_n = 0, \quad \text{on} \quad y = S. \tag{3.7}$$

It is difficult to obtain many terms of the solution series manually. So a MAPLE program has been written that calculates successively the coefficients of the solution series. It consists of the following segments:

(i) Declaration of arrays for the solution series coefficients; ψ = array (0……..25), ω = array (0……25).
(ii) Input the leading order term and their derivatives *i.e.* $\psi_{0,}\omega_{0.}$
(iii) Input the modeled channel geometry slope (*i.e.* d*S*/d*x*).
(iv) Using a MAPLE loop procedure, iterate to solve equations (3.5)-(3.7) for the higher order terms *i.e.* $\psi_{n,}\ \omega_n$, n = 1,2,3,…… .
(v) Compute the wall shear stress and the axial pressure gradient.

The first two terms of the solution for stream-function and vorticity are obtained as

$$\psi=[\frac{1}{2}(3-kS^2)\eta+\frac{1}{2}(kS^2-1)\eta^3]+\frac{\mathrm{Re}\,S}{280}[\eta(7H^2S^3k+5k^2S^4-6kS^2-7H^2S+15)-\eta^3(14H^2S^3k+11k^2S^4-16kS^2-14H^2S+33)+7\eta^5(H^2S^3k+k^2S^4-2kS^2-H^2S+3)-\eta^7(k^2S^4-4kS^2+3)]+O\left(\mathrm{Re}^2\right) \quad (3.8)$$

$$\omega=[\frac{3\eta}{S^2}(1-kS^2)]+\frac{3\,\mathrm{Re}}{140\,S}[\eta(14H^2S^3k+11k^2S^4-16kS^2-14H^2S+33)-\frac{70\eta^3}{3}(H^2S^3k+k^2S^4-2kS^2-H^2S+3)+7\eta^5(k^2S^4-4kS^2+3)]+O\left(\mathrm{Re}^2\right) \quad (3.9)$$

where $\eta = y/S$. The shear stress at the boundary of the channel is given by

$$\tau_w=-\frac{1}{1+b_x^{\,2}}\left[\left(\sigma_{yy}-\sigma_{xx}\right)b_x+\left(1-b_x^{\,2}\right)\sigma_{xy}\right] \quad \text{on } y=b(x) \quad (3.10)$$

where σ_{yy} , σ_{xx} , σ_{xy} are the usual stress components, *i.e.*,

$$\sigma_{xy}=\mu\left[\psi_{yy}-\psi_{xx}\right]$$
$$\sigma_{yy}-\sigma_{xx}=-4\mu\psi_{xy} \quad (3.11)$$

The subscripts (x, y) denote partial differentiation with respect to (x, y), respectively. The dimensionless form of wall shear stress can be written as:

$$G=\frac{a^2S^2}{\mu Q}\tau_w=-\frac{S^2}{\left(1+\varepsilon^2S_x^{\,2}\right)}\left[\left(\psi_{yy}-\varepsilon^2\psi_{xx}\right)\left(1-\varepsilon^2S_x^{\,2}\right)-4\varepsilon^2S_x\psi_{xy}\right] \quad \text{on } y=S \quad (3.12)$$

for $0<\varepsilon<<1$ we obtain

$$G=\left(3-3S^2k\right)+\mathrm{Re}\,S\left(-0.34286+0.2\,SH^2+0.05714S^2k-0.2\,S^3H^2k-0.11429S^4k^2\right)+O\left(\mathrm{Re}^2\right) \quad (3.13)$$

Internal Flow Separation

We have investigated the solution behavior by algebraic programming language (MAPLE). The first 19 coefficients for the above solution series have been obtained which represent the flow characteristics. The above series are reformed into several diagonal Pade´ approximants of order $N = M + M$ as

$$G=\sum_{i=0}^{N}f_i\,\mathrm{Re}^i=\frac{\sum_{i=0}^{M}a_i\,\mathrm{Re}^i}{\sum_{i=0}^{M}c_i\,\mathrm{Re}^i}. \quad (4.1)$$

This method fails when the denominator of the fraction is evaluated near the zeros. By equating the numerator of equation (4.1) to zero we have computed the Reynolds number at which separation occurs in the flow field (*i.e.* $G\to 0$) for different values of k and H at position S = 1on the channel, as shown in Table 1.

Table 1: Computations showing the Reynolds number for internal flow separation development in the diverging channel at $S = 1$.

$k\downarrow$	$H\rightarrow$	0.0	0.5	1.0	1.5
0.1	Re	6.93516	7.98599	12.56802	22.62489
0.01	Re	7.52575	9.12779	16.59612	29.58886
0.001	Re	7.42219	9.23728	16.96519	29.59017
0.0	Re	7.93235	9.26564	17.08937	32.97352

Results and Discussions

Since the fluid is incompressible and viscous, the above mathematical analysis is very suitable for liquid. The combined effects of homogeneous magnetic field and permeable parameter on the flow have been investigated. Flow separations have been observed at a given position in the slowly varying exponentially diverging symmetrical channel having permeable walls and computed numerically as shown in Table 1.

Figure 2 below shows the axial fluid velocity profile in the diverging channel. A parabolic axial velocity profile is observed with maximum value at the channel centerline and minimum value at the walls. At the channel centerline and its beside it is observed that the effect of increasing values of the magnetic field intensity (H) and permeable parameter (k) is to decrease the magnitude of axial velocity profile. This finding is identical to those of Makinde and Mhone (2006), who showed the axial fluid velocity profile in an exponentially diverging channel having rigid boundaries under the effect of an external magnetic field.

Figure 3 below shows the normal fluid velocity profile in the diverging channel where the maximum value is observed near the middle position of the channel centerline and the wall. But at the channel centerline and wall the minimum value of the normal velocity profile is observed. Moreover, a general increase in the magnitude of normal velocity profile is noticed with an increase in both magnetic field intensity and permeable parameter. These findings are identical to those of Makinde and Osalusi (2006), who showed the normal fluid velocity profile in a channel with permeable boundaries under the effect of external magnetic field, but they found a general decrease in the magnitude of normal velocity profile with an increase in both wall slip and magnetic field intensity.

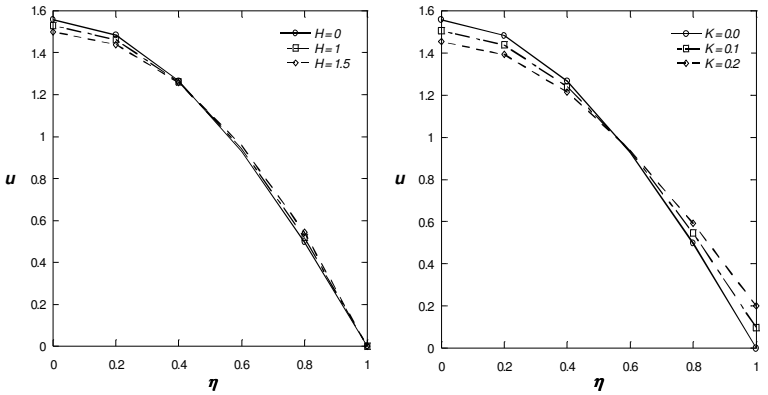

Figure 2: Axial fluid velocity profiles for different values of H and k ; $S = 1$ and Re = 1.

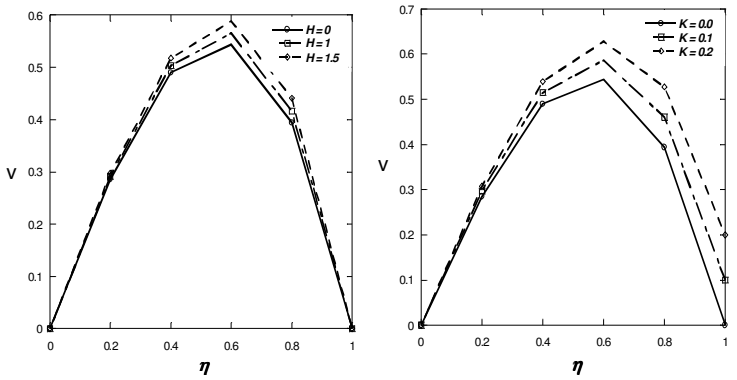

Figure 3: Normal fluid velocity profiles for different values of H and k; $S = 1$ and Re = 1.

Figures 4(a)-4(b) represent the wall shear stress (G) with respect to flow Reynolds number at $S = 1$ in the diverging channel. Generally, increased values of wall shear stress are observed for higher values of magnetic field intensity and lower values of permeable parameter. Here it is interesting to note that the requirement of flow Reynolds number for the development of internal flow separation increases as the magnetic field intensity increases and permeable parameter decreases in magnitude, which is also clear from Table 1. Meanwhile, a further increase in magnetic field intensity may suppress or totally prevent the development of internal flow separation in the diverging channel. These investigations agree with those of Makinde and Mhone (2006), who found the results for exponentially diverging channel with rigid boundaries. When $k = 0.0$ our results match with the study of them for different values of H. From Table 1 above and Figures 4(a)-4(b), it is also clear that if flow Reynolds number is sufficiently high, internal flow separation development is still possible at low magnetic field intensity. Hence, in order to prevent the occurrence of internal flow separation in the diverging channel, the imposed external magnetic field intensity on the conducting fluid must be sufficiently high and the value of the permeable parameter must be sufficiently low as well.

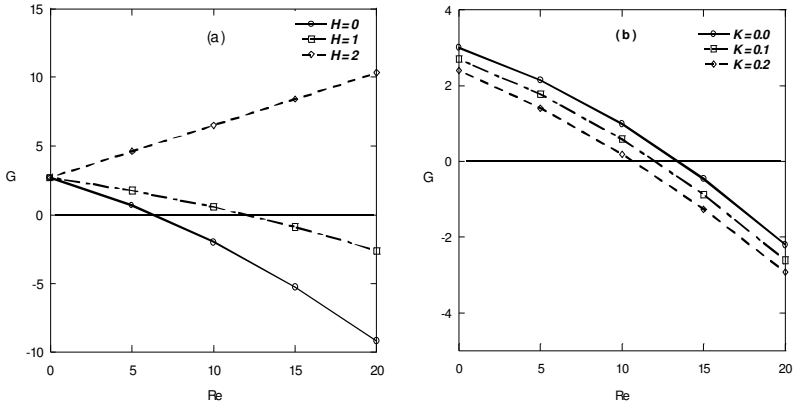

Figure 4: Wall shear stress for different values of H and k; (a) when $k = 0.1$ and (b) when $H = 1$.

Conclusion

We investigated the combined effects of homogeneous magnetic field and permeable parameter on the steady flow in a slowly varying exponentially diverging channel having permeable walls. Our results revealed that the effect of increasing values of the magnetic field intensity and permeable parameter is to decrease the magnitude of axial velocity profile around the centerline of the channel, and between the centerline and walls the normal velocity profile increases by both magnetic field and permeable parameter. We also noticed that generally early separation occurred with an increase in permeable parameter and decrease in magnetic field intensity. The number of coefficients of the solution series above 19 would give us more accurate values of the flow Reynolds number for the internal flow separation, however, both magnetic field and permeable parameter have great influence on behavior of fluid velocity profiles and development of internal flow separation in the diverging channel.

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