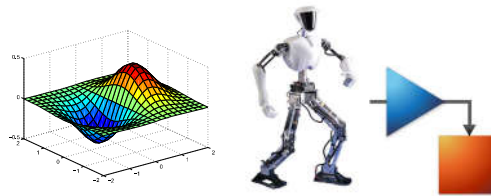


Control Engineering Sessional

Dr. Harunur Rashid
Md. Zahirul Islam
Md. Abu Jafar Rasel



Fall 2016

Gratitude

I am immensely grateful to the following person for the preparation and revision of this lecture.

- Mr. Md. Minal Nahin, Graduate Research Assistant , ME, IUPUI
- Mr. Anjan Goswami, Assistant Professor, MPE, AUST
- Dr. Md. Zahurul Haq, Professor , ME, BUET
- Dr. Sumon Saha, Assistant Professor , ME, BUET

Course Content & Syllabus

Tentative Course Plan ¹

Topic	Week ²
Plotting and Linear Equations	1-2
Polynomials and Curve Fitting	3
Math Operations and Linear Equations in Simulink	4
Differential Equations and System Response	5
Transfer Functions, Zero-Poles and Stability	6
Laplace Transform and Residue	7
System Modelling and Controllers	8
PID Controller Tuning	9
Experiment	10
Experiment	11
Experiment	12
Final Examination	13

¹with Mr. Abu Jafar Rasel

²Each lecture is approximately 120 min.

Marks Distribution

- ① Quizzes 50%
- ② Classwork 40%
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- 9 **For rough calculations you will need to use MATLAB command window.**

Introduction

- **Control engineering** or **control systems engineering** is the engineering discipline that applies control theory to design systems with desired behaviors.
- The practice uses sensors to **measure** the output performance of the device being controlled and those measurements can be used to give **feedback** to the input **actuators** that can make corrections toward desired performance.
- When a device is designed to perform without the need of human inputs for correction it is called **automatic control** (such as cruise control for regulating a car's speed).
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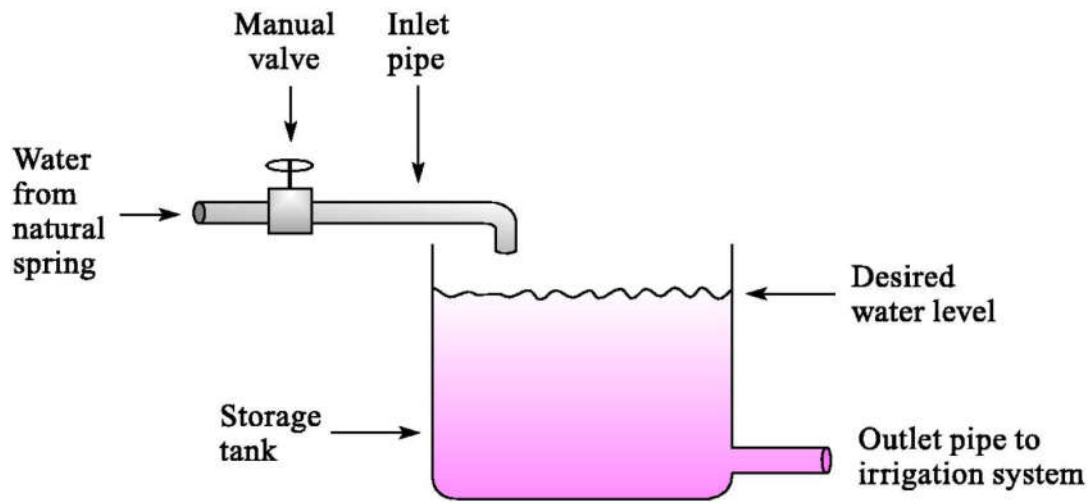
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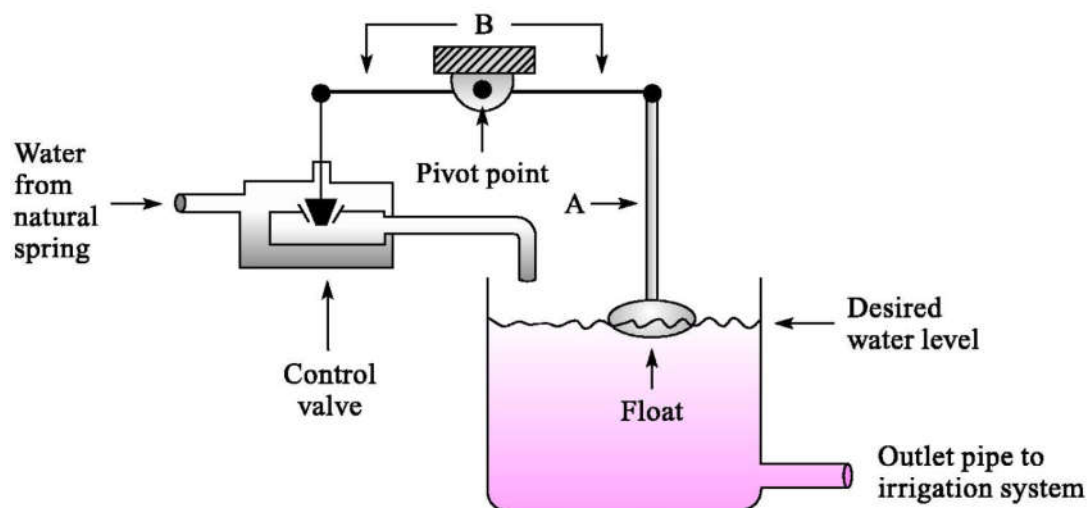
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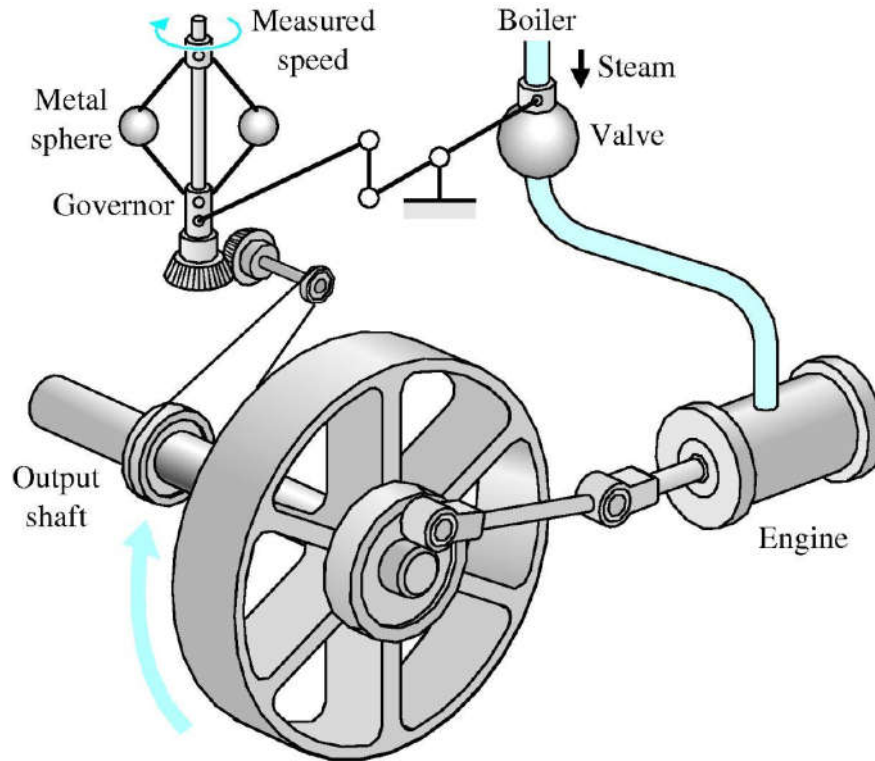
Liquid Level Control



Automatic Control

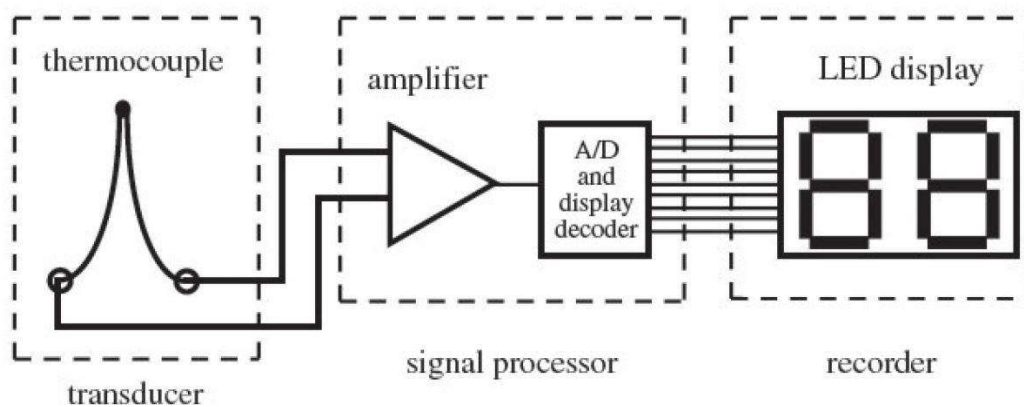


Watt's Flyball Governor



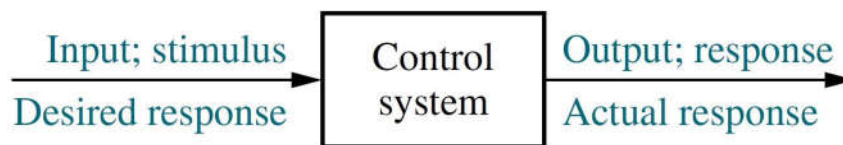
Elements of Measurement System

Consists of 3 basic elements



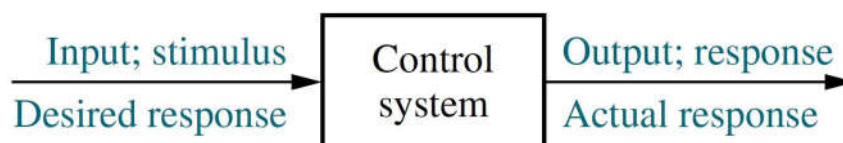
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Classification of Control Systems



Toaster



Air Conditioner

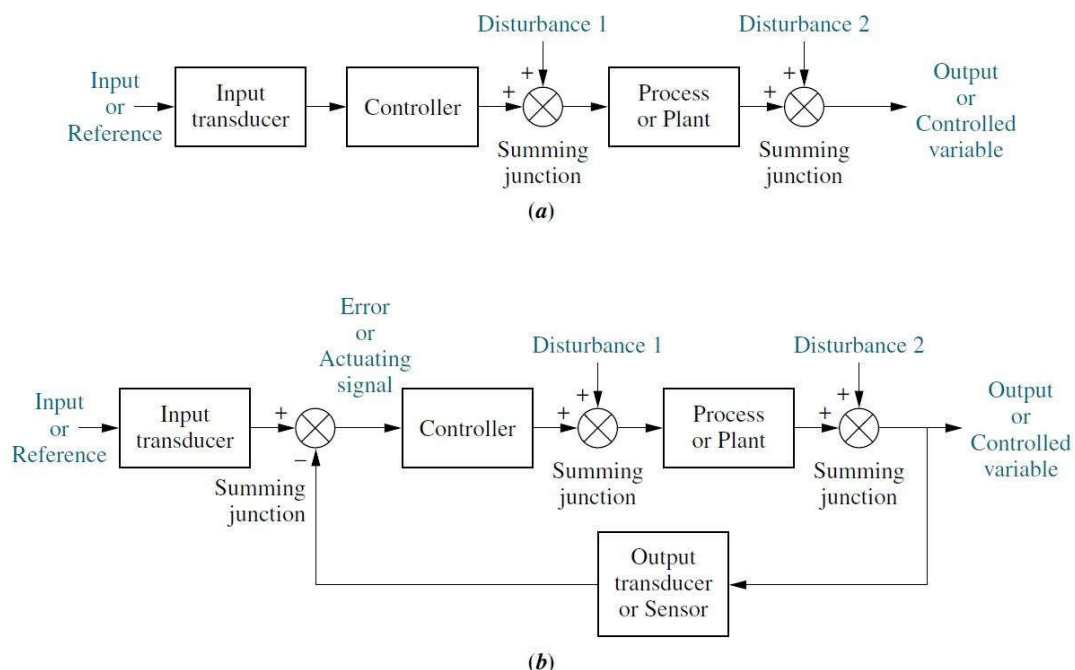
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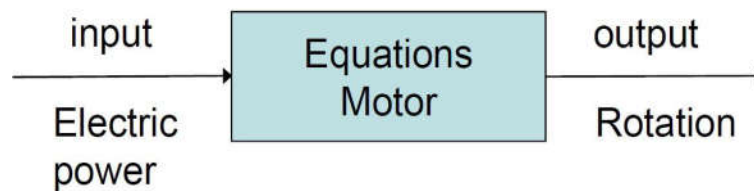
Classification of Control Systems



Block diagrams of control systems: **a.** open-loop system; **b.** closed-loop system

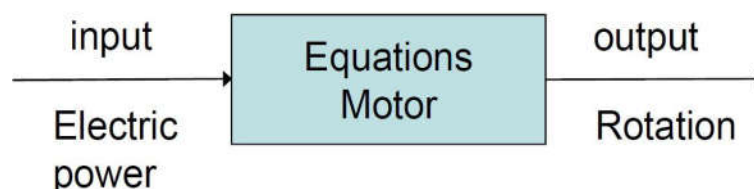
Definition and Modeling of Systems

- A **system** can be thought of as a box which has an input and an output.
- **Equations** are used to describe the relationship between the input and output of a system.
- **Response** of a system is a measure of its fidelity to its purpose.
- **Modeling** is the process of representing the behavior of a system by a collection of mathematical equations & logics.
- **Simulation** is the process of solving the model and it is performed on a computer.
- In this course, we're going to learn to simulate systems using **MATLAB-Simulink**



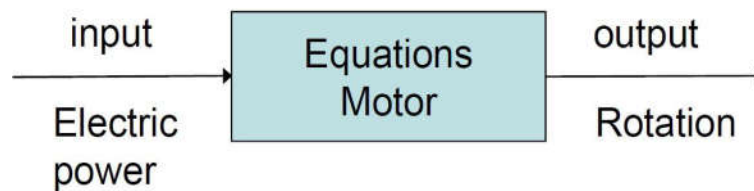
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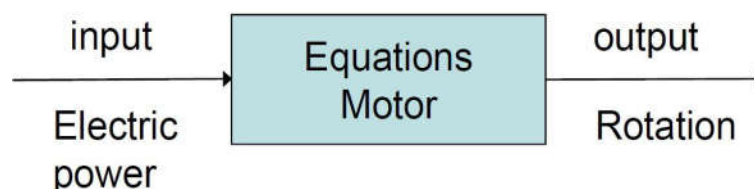
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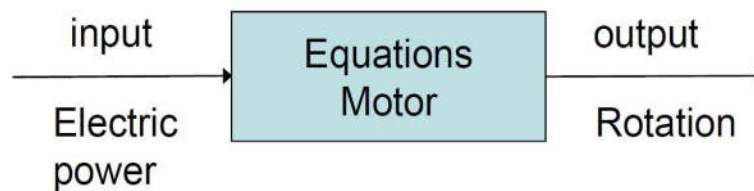
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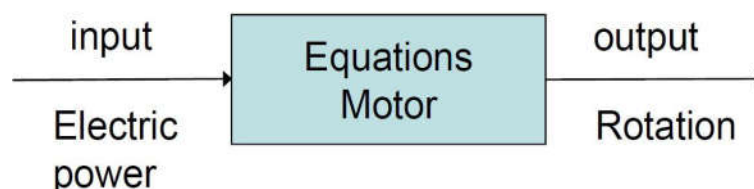
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- It is primarily used to perform scientific computation and visualization.
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 - ① Command Window: *To execute commands.*
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Try **mesh(X,Y,Z)** instead of surf(X,Y,Z)

Practice

Plot the function $c = a^2 - b^2$ over the range $-2 \leq a \leq 2$ and $-2 \leq b \leq 2$

Plot the function $y = \sin x$ over the range $-2\pi \leq x \leq 2\pi$

Plot the function $y = x^3 - x^2 + 3$ over the range $-4 \leq x \leq 4$

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- 9 **area(x,y)** ⇒ same as **plot(x,y)** but the area under the curves are filled.

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- 6 **shading interp** \Rightarrow Changes shading of plot(faceted, flat and interp).
- 7 **hold on** \Rightarrow holds the current figure window for further plotting.
- 8 **ezplot('x')** \Rightarrow plots the graph of the string input function
- 9 **area(x,y)** \Rightarrow same as **plot(x,y)** but the area under the curves are filled.
- 10 **bar(x,y)** \Rightarrow plots bar graph .

Some useful functions for plotting

- 1 **figure** \Rightarrow Creates an empty figure window.
- 2 **title('string')** \Rightarrow Creates title in figure window.
- 3 **xlabel('x-axis')** \Rightarrow Creates x-axis label in figure window.
- 4 **ylabel('y-axis')** \Rightarrow Creates y-axis label in figure window.
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- 10 **bar(x,y)** \Rightarrow plots bar graph .
- 11 **mesh(x,y,z)** \Rightarrow same as **surf(x,y,z)** except patches between lines are not filled.

Solving Linear Equations : *linsolve* function

Any linear system of equations must have 0, 1 or infinite number of solutions.

linsolve(A,B) takes coefficient matrix A and constant matrix B as input and solves the system.

Solve the following system.

$$\text{System of Linear equations } \begin{cases} x + 3y + z = 5 \\ 2x - y + 2z = 17 \\ 3x + 4y + 5z = 26 \end{cases}$$

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Solution visualization

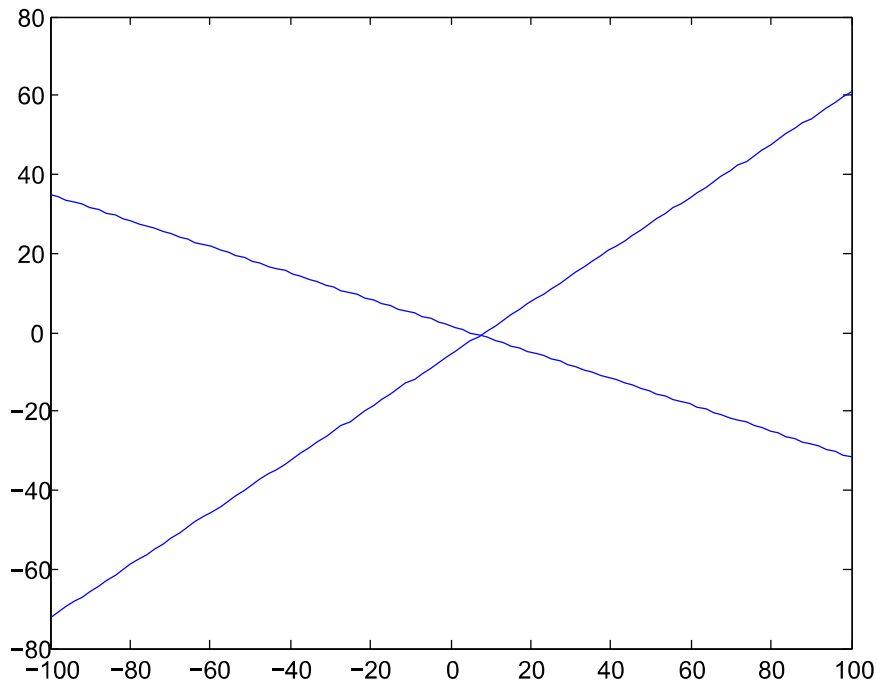
Draw the plot of the following equations in same graph.

$$\text{System of Linear equations with one solution } \begin{cases} x + 3y = 5 \\ 2x - y = 17 \end{cases}$$

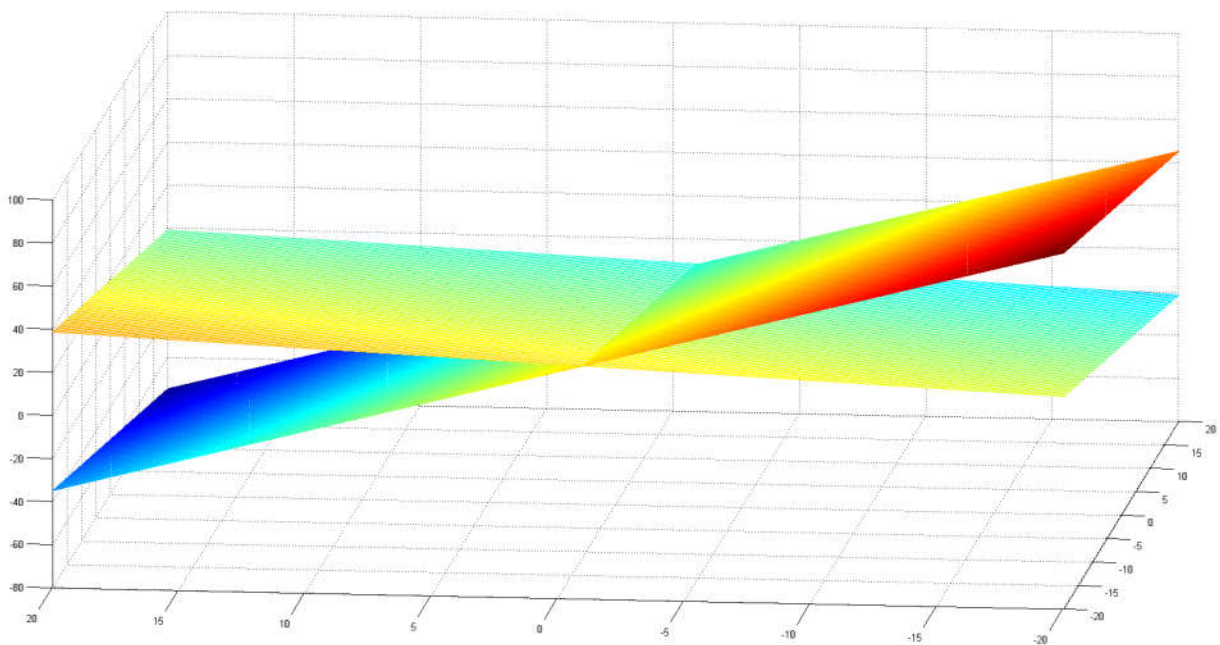
Draw the mesh plot of the following equations in same graph.

$$\text{System of Linear equations with infinite solutions } \begin{cases} x + 3y + z = 5 \\ 2x - y + 2z = 17 \end{cases}$$

Solution visualization (contd.)



Solution visualization (contd.)



Polynomials

- **MATLAB** interprets an **nth** order polynomial as a row vector of $n + 1$
- Consider a polynomial $P(s) = s^4 + 3s^3 - 15s^2 - 2s + 9$
- Enter it in MATLAB as row vector P as follows
- $P = [1 \ 3 \ -15 \ -2 \ 9]$ ←
- or $P = [1, 3, -15, -2, 9]$ ←
- To evaluate the value of P at $s = 3$ or $P(3)$ use the following command **polyval(P,s)**
- **polyval(P,3)** ←
- MATLAB will show you the value of $P(3)$ to be 30.
- If there are missing terms, zeros must be entered at the appropriate places.

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Practice

Evaluate the value of
 $y = 2s^2 + 3s + 4$ at $s = 1, -3$

Practice

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Answer: $y(1) = 9$ $y(-3) = 13$

Practice

Evaluate the value of
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Practice

Evaluate the value of

$$y = 2s^3 + 1 \text{ at } s = -4$$

$$\text{Answer: } y(-4) = -127$$

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Answer: $-2, -1$

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Find the polynomial whose roots are -5.5745,2.5836,-0.7951,0.7860 .

Answer: $P(s) = s^4 + 3s^3 - 15s^2 - 2s + 9$

Conv(x,y) Function: Convolution

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Multiply $(s + 2)$ and $s^2 + 4s + 8$

Answer: $s^3 + 6s^2 + 16s + 16$

Practice

Evaluate the product of $(s + 3)$,
 $(s + 6)$ and $(s + 2)$

Practice

Evaluate the product of $(s + 3)$, $(s + 6)$ and $(s + 2)$

Answer: $s^3 + 11s^2 + 36s + 36$

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Practice

Evaluate the division of
 $(s^2 - 1)$ by $(s + 1)$

Practice

Evaluate the division of
 $(s^2 - 1)$ by $(s + 1)$

Answer: $(s - 1) + 0$

Curve fitting: polyfit function

From Data to Equation.

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- k specifies the order of the desired polynomial.
- The result c will be a row vector containing the coefficients of the polynomial.

Practice

Find a second order polynomial to predict the discharge(Q) of a pump after 1.5 seconds from the given data.

t(s)	0	1	2	4
Q(L/s)	1	6	20	100

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t(s)	0	1	2	4
Q(L/s)	1	6	20	100

Answer:

- 1 $t=[0 \ 1 \ 2 \ 4];$
- 2 $Q=[1 \ 6 \ 20 \ 100];$
- 3 $c=\text{polyfit}(t,Q,2)$
- 4 MATLAB returns, $c = 7.3409 \ -4.8409 \ 1.6818$
- 5 $\therefore Q = 7.3409t^2 - 4.8409t + 1.6818$ is the equation of discharge.
- 6 Use **polyval(c,1.5)** to get the answer 10.9375 L/s.

Advanced Polynomial Operations

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Practice this commands using the polynomial $s^4 + 4s^3 + 8s^2 + 16$

SIMULINK

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- It is very useful for modeling, simulating and analyzing dynamic systems due to its GUI.
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Basic Math Operations

Demonstration

Scope Vs. Display

Demonstration

- ① Add two numbers and check the output in both Scope and Display
- ② Subtract two numbers and check the output in both Scope and Display
- ③ Multiply two numbers and check the output in Scope and Display
- ④ Divide two numbers and check the output in Scope and Display

Demonstration

View the response of $50 + 60\sin(0.25t + \phi)$

Demonstration

View the response of $50 + 60\sin(0.25t + \phi)$ using 'add' and 'gain' operator.

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Mux

Show the three input in a single graph.

- $100 + \sin(0.25t + 2)$
- $50 + \sin(0.5t)$
- $75 + \sin(0.75t + 1)$

System of Equations

- **Simulink** can solve both linear and nonlinear system of equations.

$$\text{System of equations } \begin{cases} 2x + 3y = 13 \\ 5x - y = 7 \end{cases}$$

System of Equations

Write the equations in the following form:

$$2x + 3y = 13$$

$$\Rightarrow x = \frac{13 - 3y}{2}$$

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System of Equations

Write the equations in the following form:

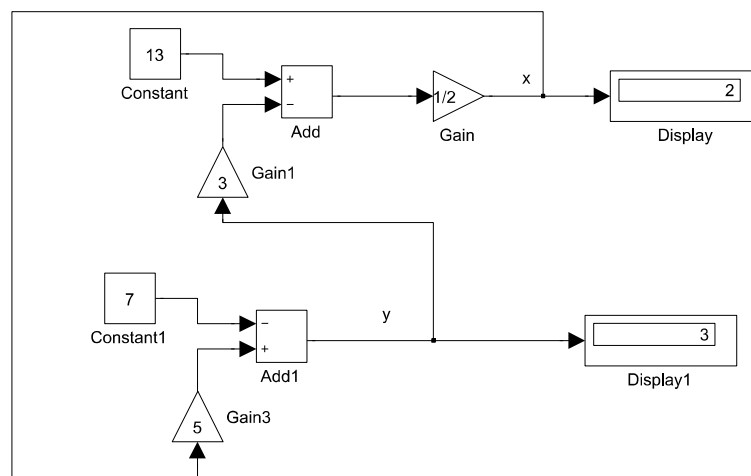
$$2x + 3y = 13$$

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System of Equations



Classwork

Solve the following system.

$$\text{System 1} \begin{cases} x + 3y + z = 5 \\ 2x - y + 2z = 17 \\ 3x + 4y + 5z = 26 \end{cases}$$

Classwork

Solve the following system.

$$\text{System 2} \begin{cases} a + b + c = 6 \\ 2a + 3b + 4c = 15 \\ a + 2b - c = 10 \end{cases}$$

Solution of ODE

Demonstration

$$\text{If } \frac{dy}{dt} = 0.5t^2 \text{ find } y.$$

$$y(0) = 0$$

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$$\frac{dx}{dt} - 3t^2 = 2t + \sin(0.25t) ; x(0) = 0$$

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Order of a system is designated by order of the differential equation.

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- Example: Potentiometer, ideal spring etc.

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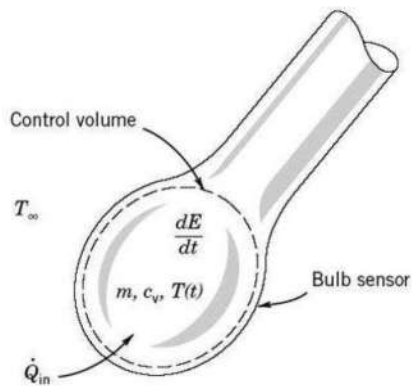
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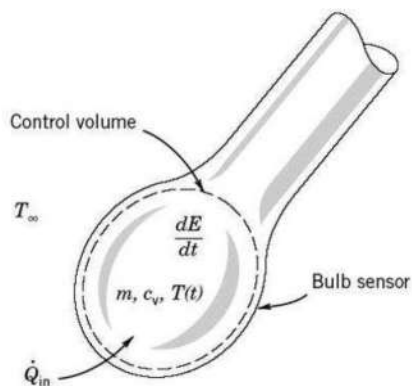
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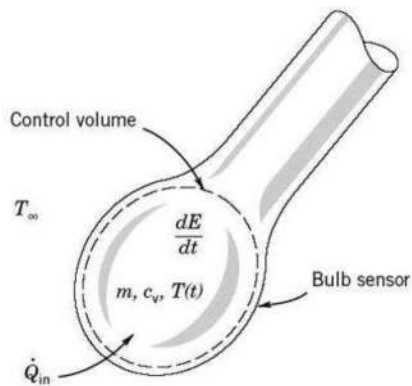


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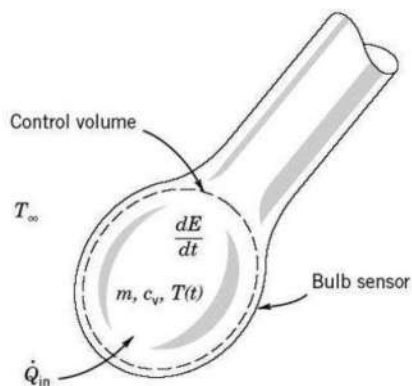
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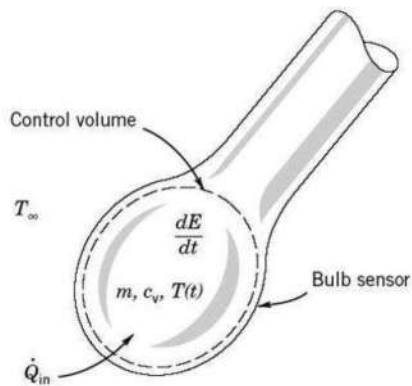
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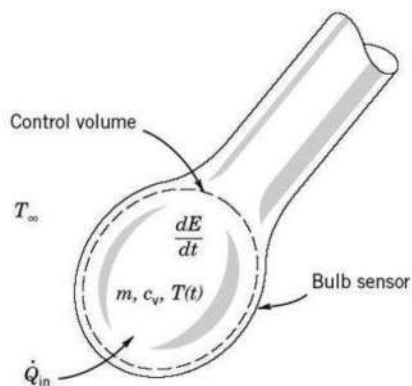
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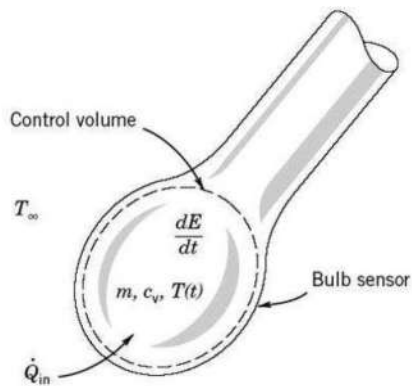
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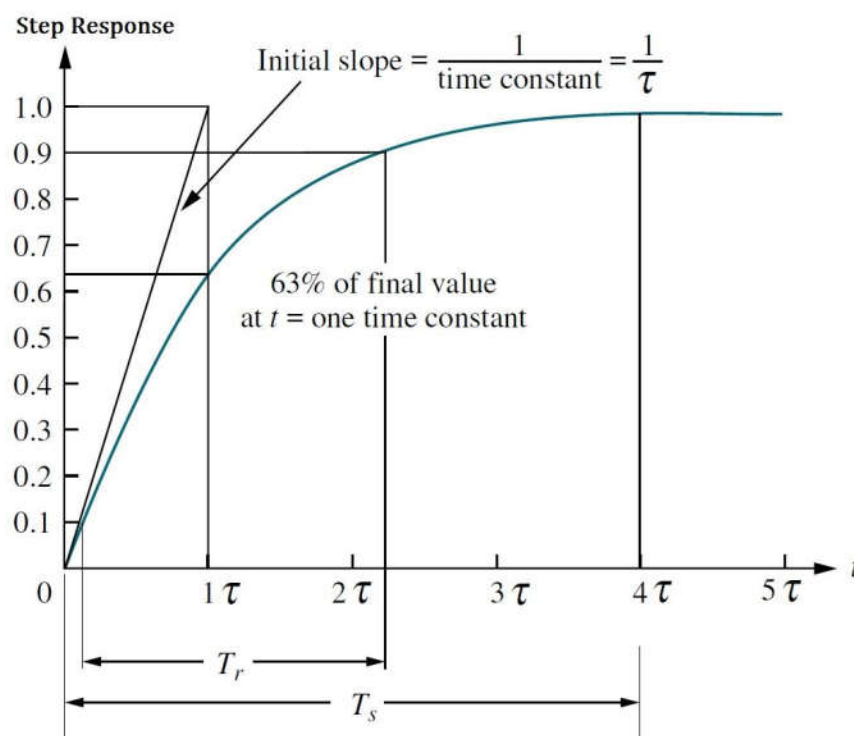
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Response - 1st Order System: Step Input



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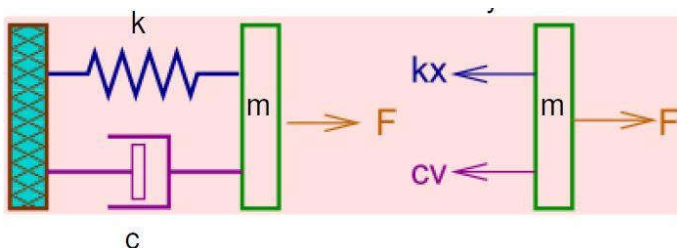
$$\Rightarrow (\tau s + 1)X(s) = kF(s)$$

$$\Rightarrow \frac{X(s)}{F(s)} = G(s) = \frac{k}{(\tau s + 1)}$$

Second Order System

Write Newton's 2nd Law for the system.

$$\sum F = ma$$

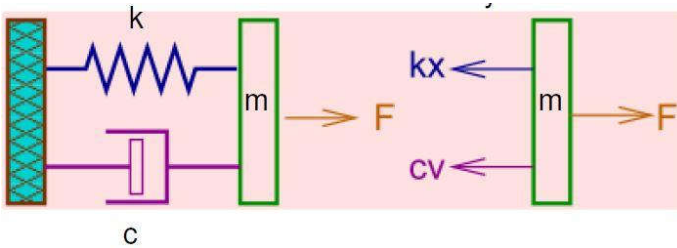


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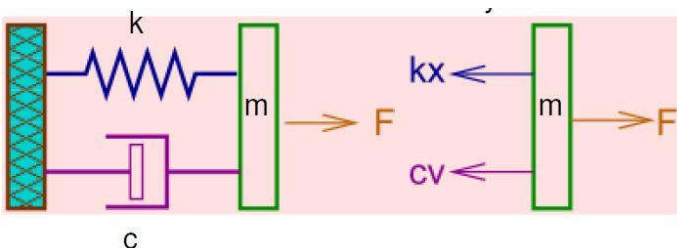
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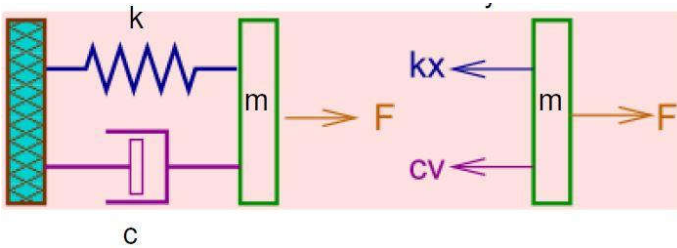
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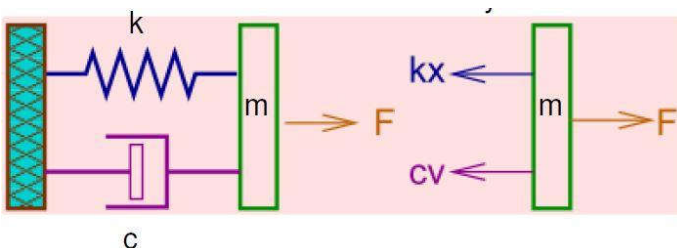
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Substitute the following:

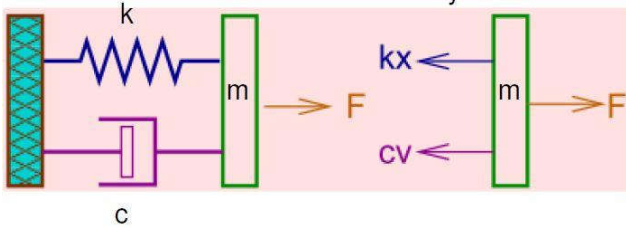
$$\omega_n = \sqrt{k/m} = \text{Undamped Natural Frequency (rad/s)}$$

$$C_c = 2\sqrt{mk} = \text{Critical Damping Coefficient}$$

$$\zeta = \frac{c}{C_c} = \text{Damping Ratio}$$

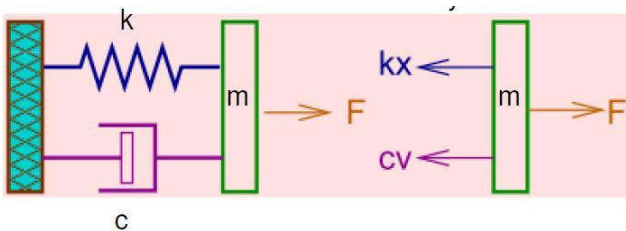
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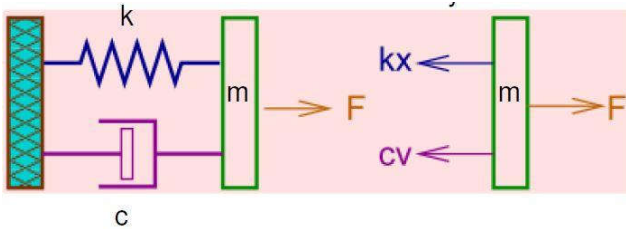
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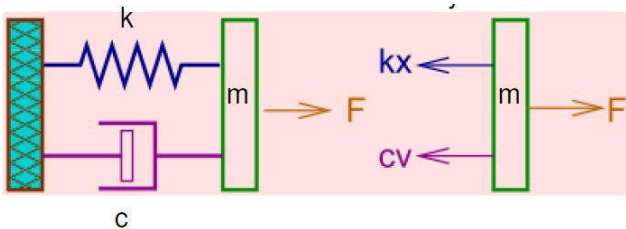
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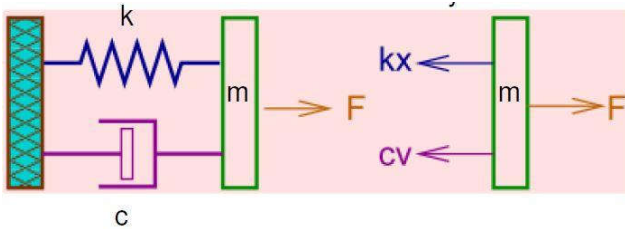
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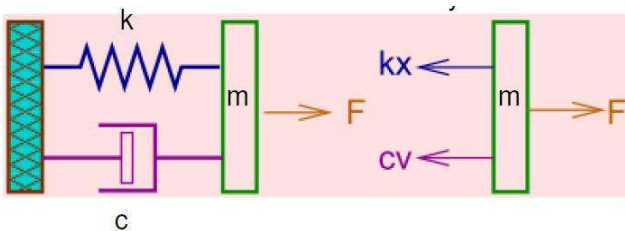
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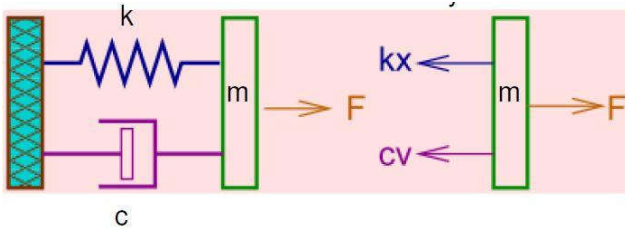
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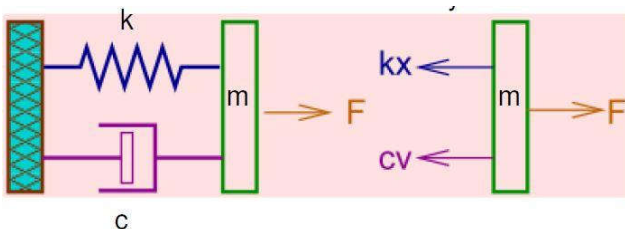
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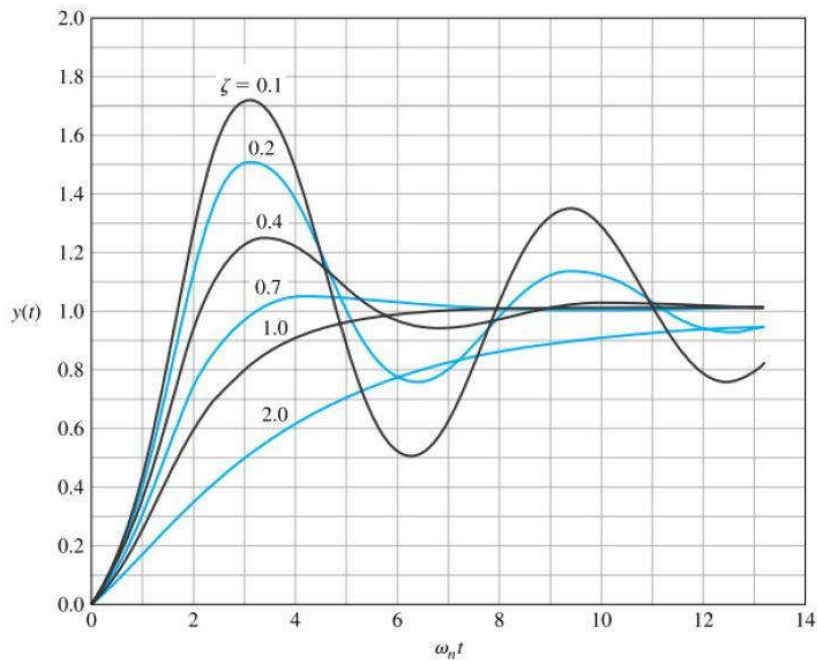
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Check the critically damped system response.

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Answer:

- 1 $n = [5 \ 15 \ 10]$
- 2 $d = [1 \ 7 \ 20 \ 24 \ 0]$
- 3 $sys = tf(n,d)$

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Practice separating the numerator and Denominator of $G(s) = \frac{5s^2+15s+10}{s^4+7s^3+20s^2+24s}$

Practice

Define a transfer function,

$$G(s) = \frac{s(s+1)(s+2)}{s(s+3)(s^2+4s+8)}$$

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Hint: Use poly and conv functions

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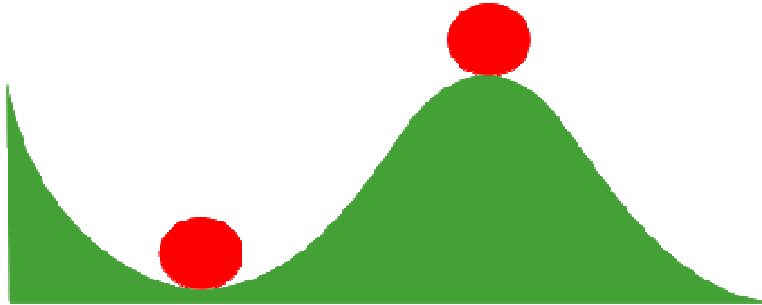
Define a transfer function, $G(s) = \frac{s+1}{s-4}$

Answer:

- 1 `s = tf('s')`
- 2 `G = (s+1)/(s-4)`

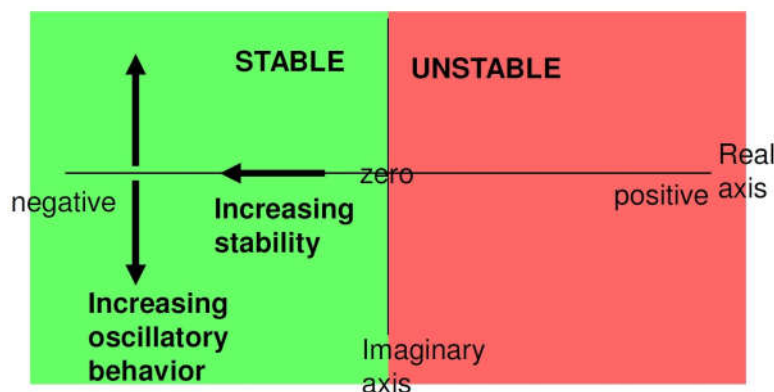
Stability Criteria

A necessary and sufficient condition for a feedback system to be stable is that all the poles of the system transfer function have negative real parts.



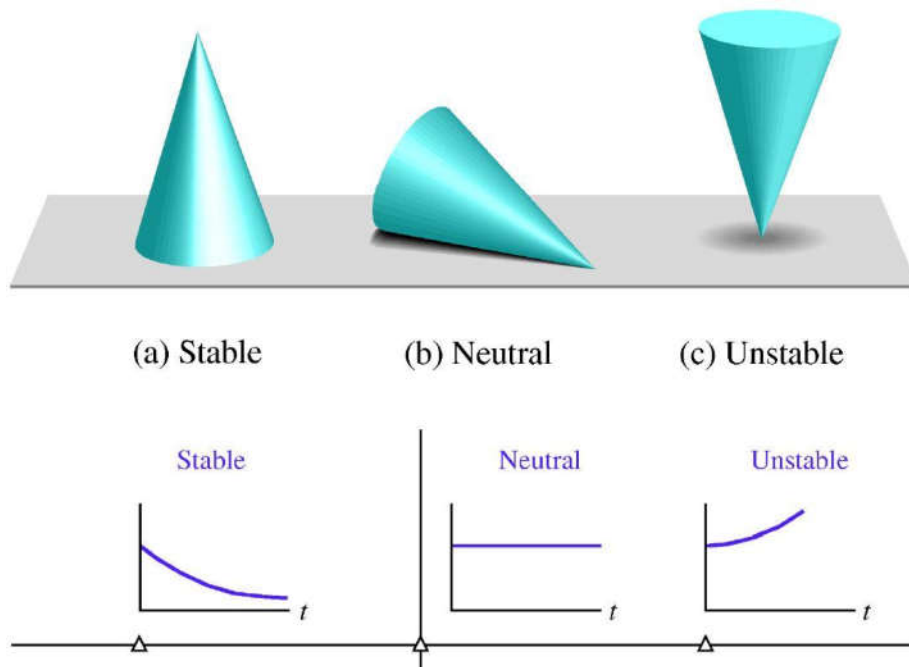
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Stability

A stable system is a dynamic system with a bounded response to a bounded input. (BIBO)



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- **[n,d] = zp2tf(z,p,k);**
- The command **pzmap(n,d)** plots the **pole-zero map** of a given transfer function.

Practice

Zero, Pole, Pole-zero map, Stability

Find the location of zeros and poles and plot the pole-zero map

of , $G(s) = \frac{2s^3+8s+6}{s^4+6s^3+12s^2+24s}$

Is the system stable?

Practice

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of , $G(s) = \frac{2s^3+8s+6}{s^4+6s^3+12s^2+24s}$

Is the system stable?

Answer: In column matrix form,

$z = -3, -1, p = 0, -4.5198, -0.7401 \pm 2.1822i, k = 2$. No.

Some useful functions for control systems

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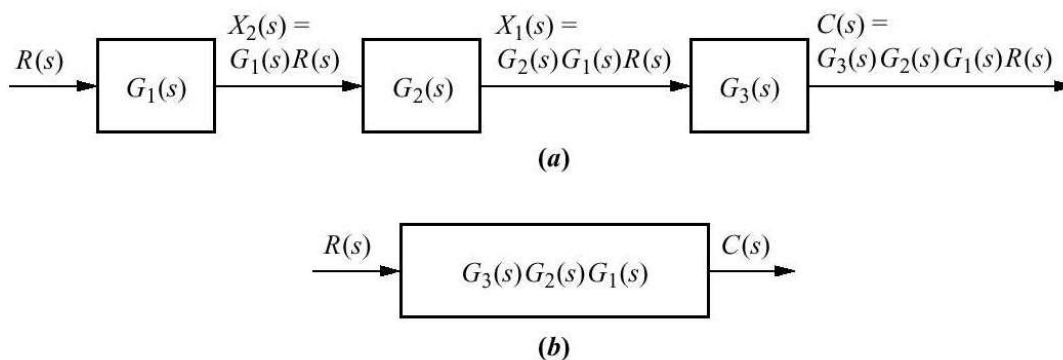
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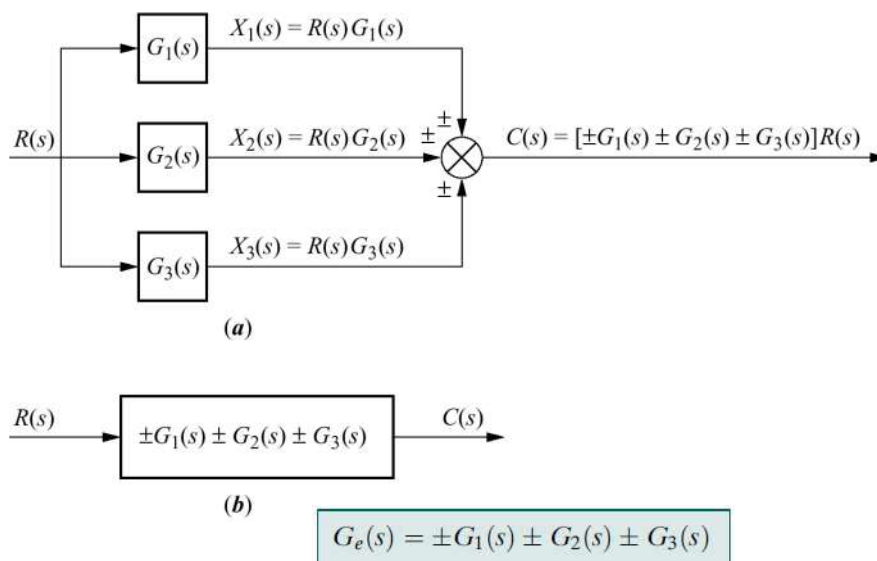
TF manipulation: Series



$$G_e(s) = G_3(s)G_2(s)G_1(s)$$

Homework: Take $G = \frac{1}{s+1}$ and $H = \frac{1}{s-1}$ to check **series(G,H)**.

TF manipulation: Parallel



Homework: Take $G = \frac{1}{s+1}$ and $H = \frac{1}{s-1}$ to check **parallel(G,H)**.

TF manipulation: Feedback

Homework: Consider,

$$G = \frac{1}{s+1}$$

$$H = \frac{1}{s-1}$$

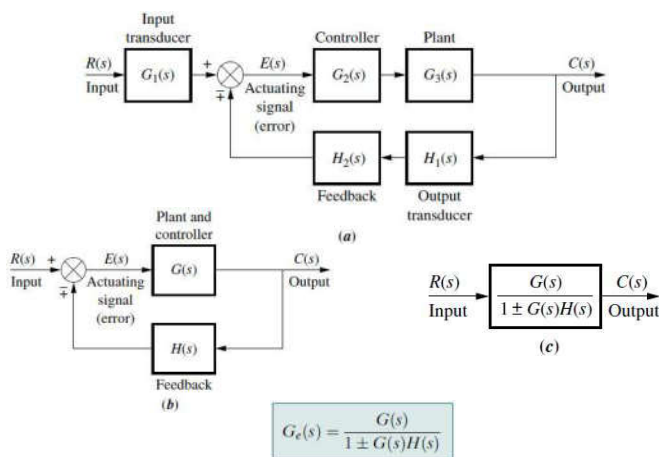


Figure: a) Feedback Control System.
b) Simplified Model.
c) Equivalent Transfer Function.

- Negative feedback system:

$$G_{eqv} = \frac{G}{1+GH} \text{ use } \mathbf{feedback(G,H)^a}$$

- Positive feedback system:

$$G_{eqv} = \frac{G}{1-GH} \text{ use } \mathbf{feedback(G,H,+1)}$$

**Closed Loop System H=1

^asame as feedback(G,H,-1)

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i.e.,

$$F(s) = \frac{1}{s+1} - \frac{1}{(s+1)^2+1}$$

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∴ the partial fraction expansion is

$$F(s) = \frac{s^3 + 9s + 1}{s^3 + s^2 + 2s + 2} = 1 + \frac{1.0000 - 1.7678i}{s - 1.4142i} + \frac{1.0000 + 1.7678i}{s + 1.4142i} - \frac{3}{s + 1}$$

Practice

Determine the Partial Fraction expansion of $F(s) = \frac{s+1}{s^4+7s^3+16s^2+12s}$

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Answer: $F(s) = \frac{0.6667}{s+3} + \frac{-0.75}{s+2} + \frac{0.5}{(s+2)^2} + \frac{0.0833}{s+0}$

Reverse Residue

$$F(s) = 1 + \frac{1.0000 - 1.7678i}{s - 1.4142i} + \frac{1.0000 + 1.7678i}{s + 1.4142i} - \frac{3}{s + 1}$$

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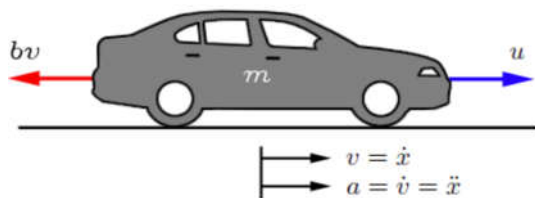
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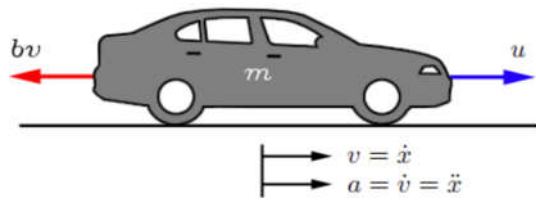
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Car Cruise Control



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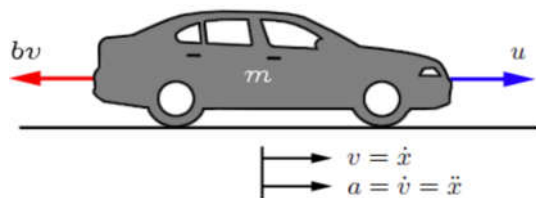
Physical Parameters

u = force generated between the road/tire interface = 500 N

b = rolling resistance proportionality constant = 50 Ns/m

m = mass of the car = 1000 kg

Car Cruise Control



Consider, a simple cruise control system with the assumption that rolling resistance and air drag are proportional to the car's speed (v).

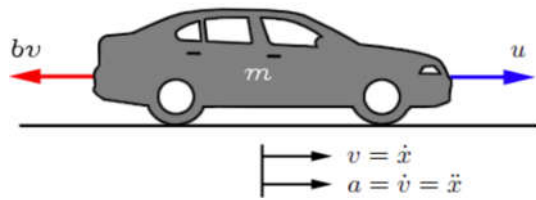
Physical Parameters

u = force generated between the road/tire interface = 500 N

b = rolling resistance proportionality constant = 50 Ns/m

m = mass of the car = 1000 kg

Car Cruise Control



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- Newton's 2nd Law : $\sum F = ma$

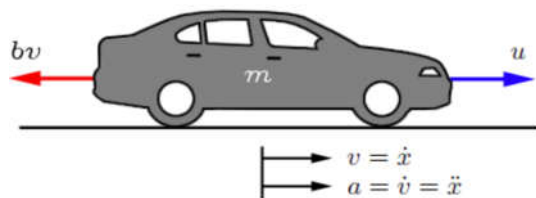
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Car Cruise Control



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- u is the force generated between the road/tire interface and can be controlled directly.

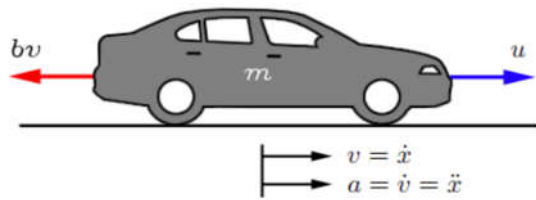
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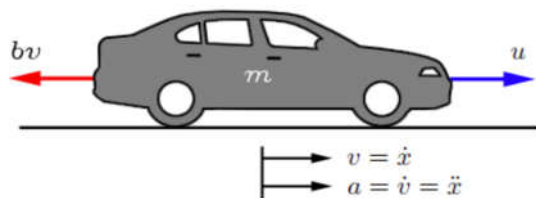
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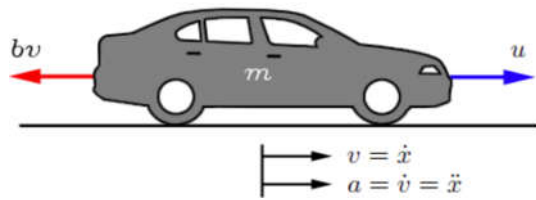
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Car Cruise Control



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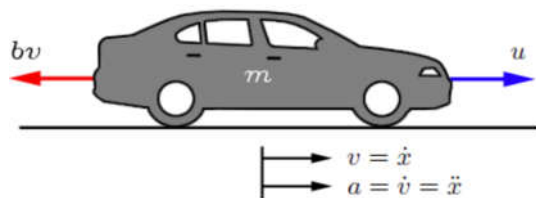
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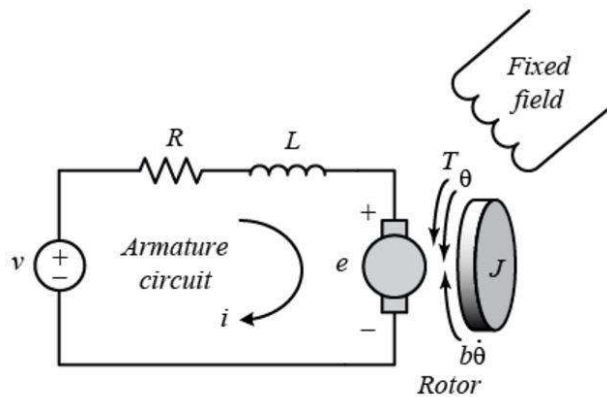
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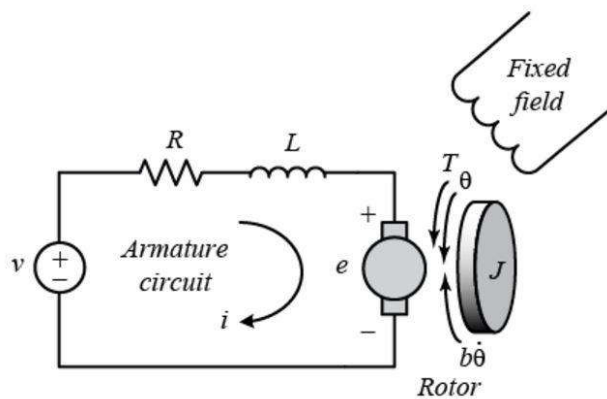
$$\therefore \frac{dv}{dt} = \frac{1}{m} \times (u - bv)$$

Response of a DC motor



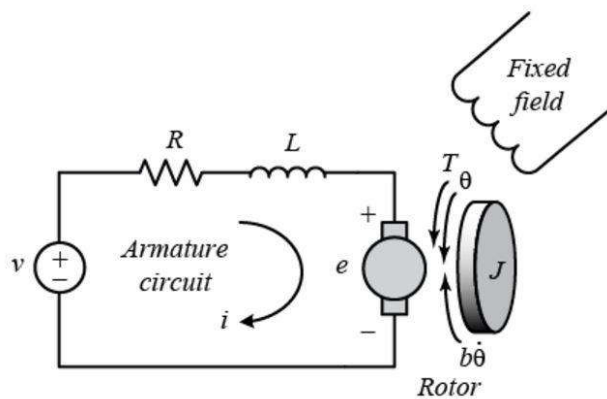
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Response of a DC motor



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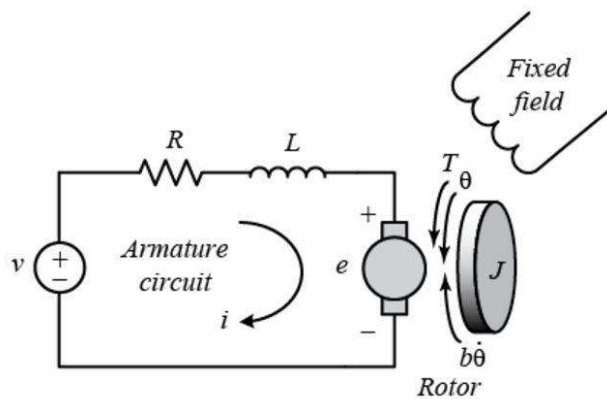
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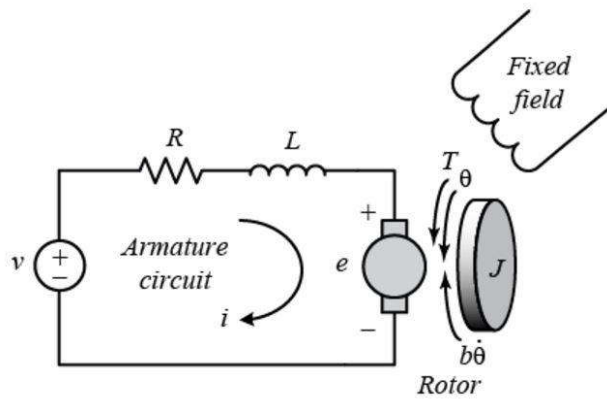
J = moment of inertia of the rotor (kgm^2)
 b = motor viscous friction constant (Nms)
 K_e = electromotive force constant (V/rad/sec)
 K_t = motor torque constant (Nm/Amp)
 R = electric resistance (Ohm)
 L = electric inductance (H)

Response of a DC motor(contd.)

From Newton's 2nd Law of Motion, $\sum F = ma$

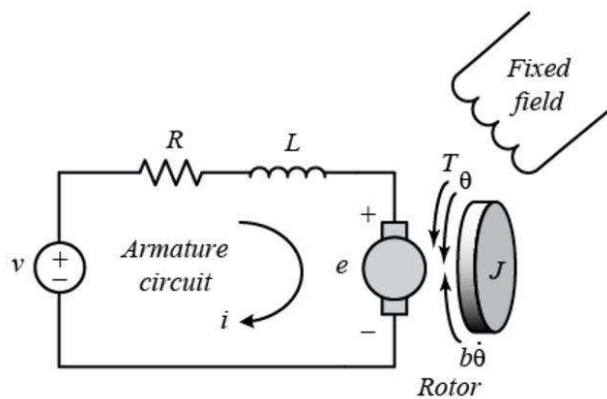


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From Newton's 2nd Law of Motion, $\sum F = ma$
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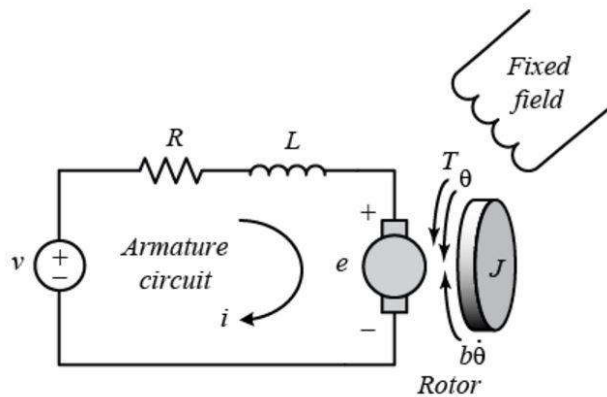
Response of a DC motor(contd.)



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Response of a DC motor(contd.)

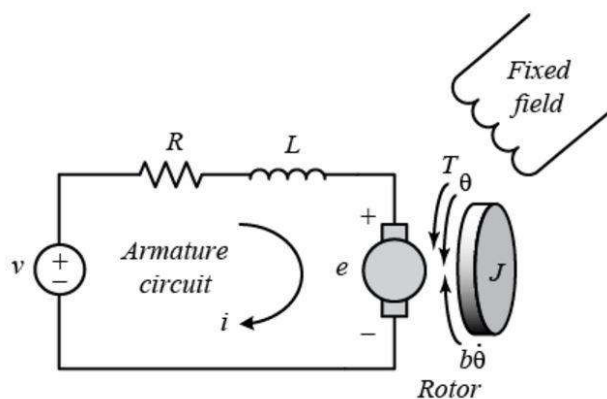


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Motor Torque is proportional to Armature Current.

Response of a DC motor(contd.)



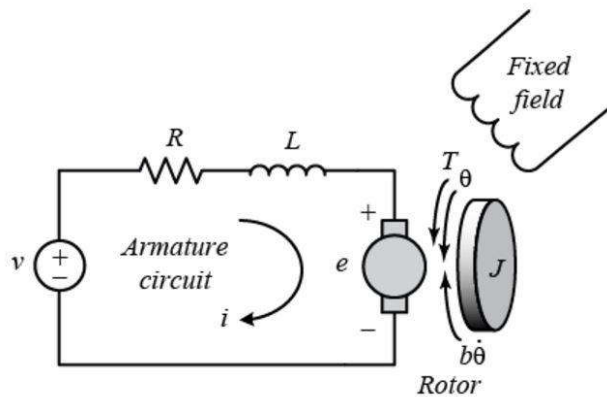
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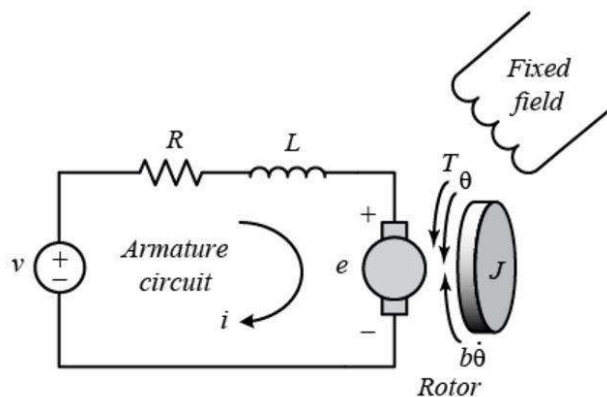
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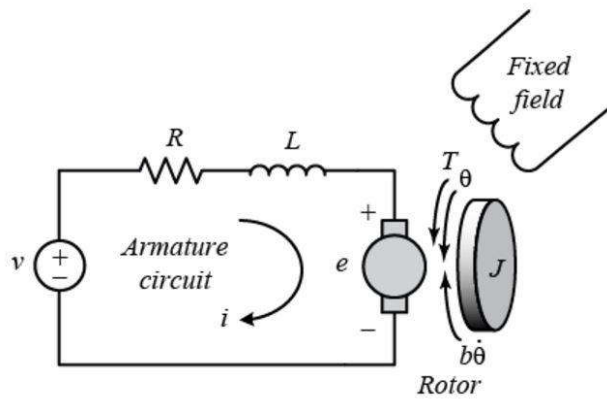
$$\therefore T = K_t i$$

$$\Rightarrow K_t i - b \frac{d\theta}{dt} = J \frac{d^2\theta}{dt^2}$$

$$\therefore \frac{d^2\theta}{dt^2} = \frac{1}{J} \left(K_t i - b \frac{d\theta}{dt} \right) \quad (1)$$

Response of a DC motor(contd.)

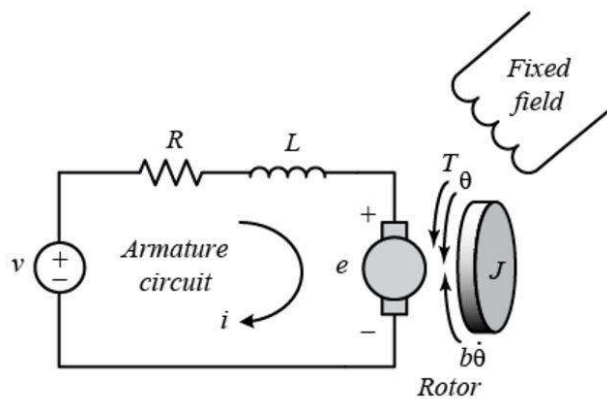
From Kirchoff's Voltage Law, $\sum V = 0$



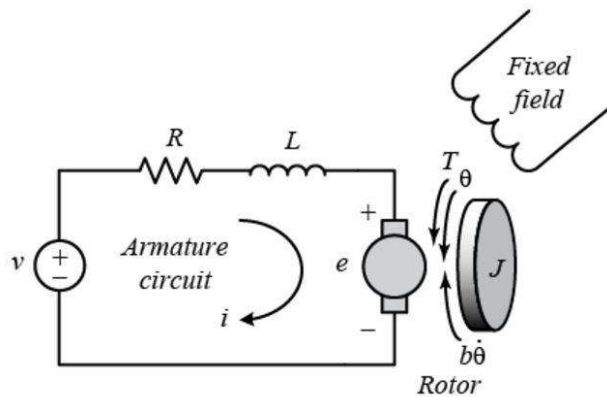
Response of a DC motor(contd.)

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$$\Rightarrow V = L \frac{di}{dt} + iR + e$$



Response of a DC motor(contd.)

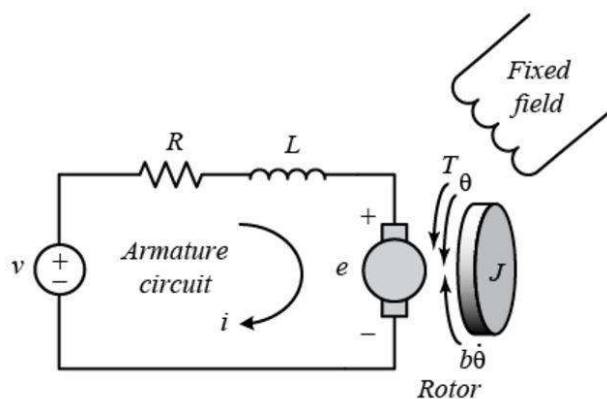


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Back emf(e), is proportional to the angular velocity of shaft.

Response of a DC motor(contd.)



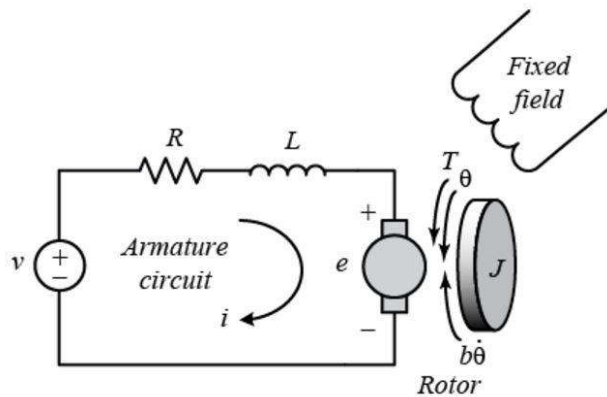
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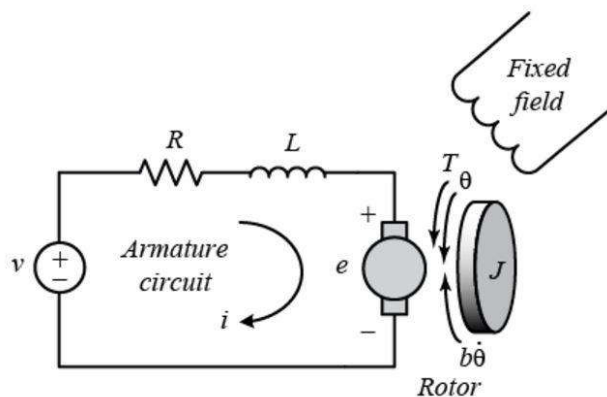
$$\Rightarrow V = L \frac{di}{dt} + iR + e$$

Back emf(e), is proportional to the angular velocity of shaft.

$$\therefore e = K_e \frac{d\theta}{dt}$$

$$\Rightarrow L \frac{di}{dt} = V - iR - K_e \frac{d\theta}{dt}$$

Response of a DC motor(contd.)



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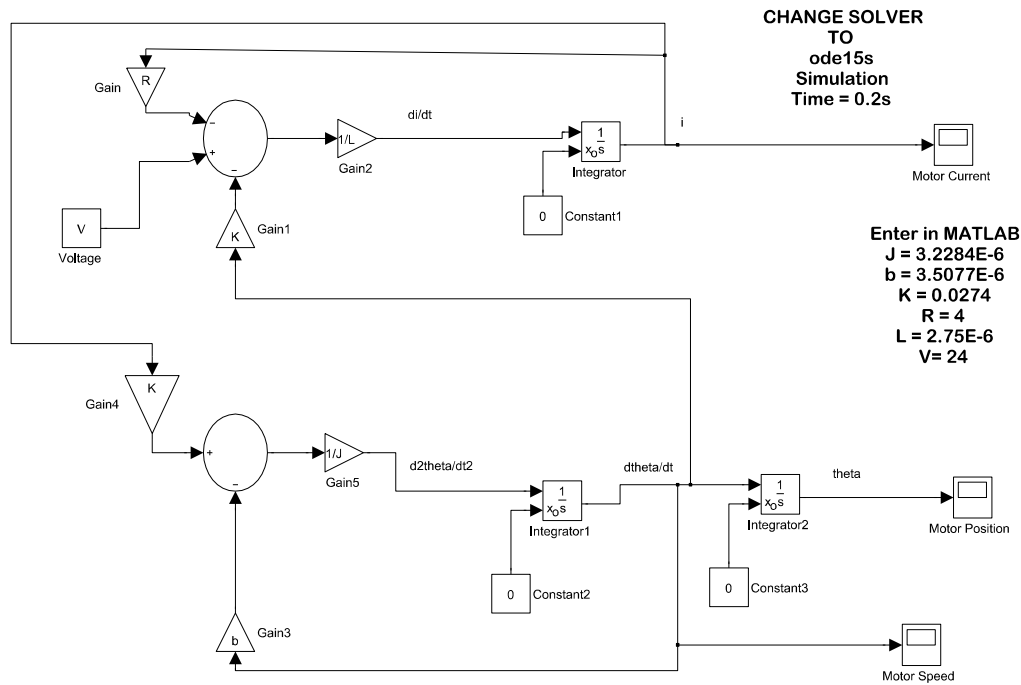
$$\therefore e = K_e \frac{d\theta}{dt}$$

$$\Rightarrow L \frac{di}{dt} = V - iR - K_e \frac{d\theta}{dt}$$

$$\therefore \frac{di}{dt} = \frac{1}{L} (V - iR - K_e \frac{d\theta}{dt}) \quad (2)$$

Solve equation (1) and (2) simultaneously in Simulink for speed/position of motor rotor.

DC Motor Simulink Model



DC Motor Simulink Model

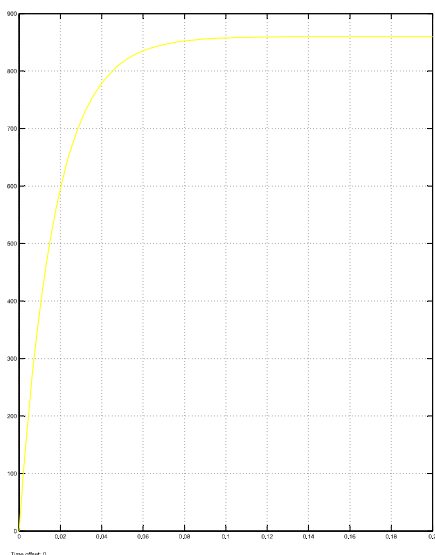


Figure: Motor Speed

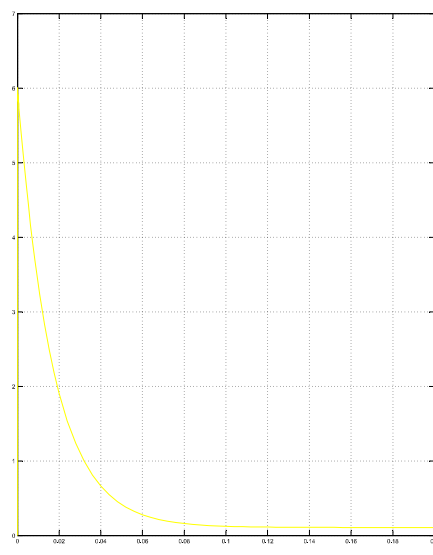


Figure: Motor Current

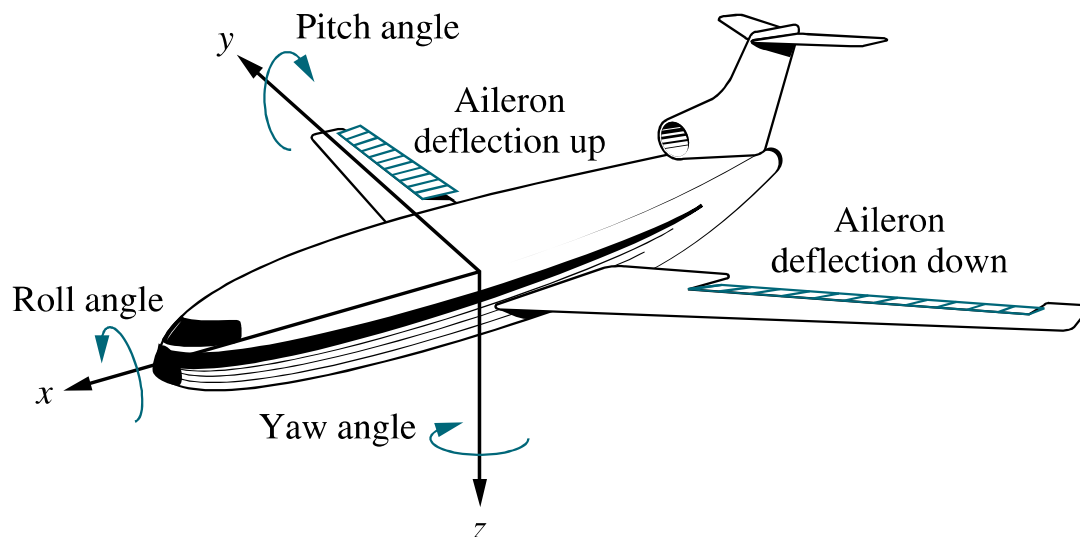
Estimating TF (Motor Speed)

Use the following data to estimate TF of motor (I=Voltage, O=Speed).

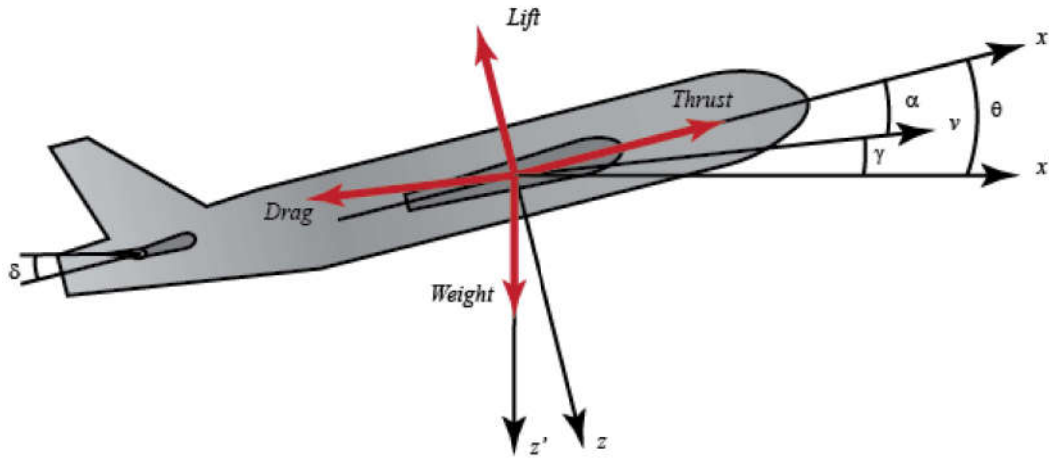
Parameter	Value
Moment of inertia of the rotor	0.01 kgm^2
Motor viscous friction constant	0.1 Nms
Electromotive force constant	0.01 $Vradsec$
Motor torque constant	0.01 $NmAmp$
Electric resistance	1 Ohm
Electric inductance	0.5 H

- 1 Declare Subsystem.
- 2 **Set Input and output linearization points. Go to Tools/Control Design/Linear Analysis Tool.**
- 3 Linearize Model. Export model to workspace from LTI viewer.
- 4 Use the command `zpk(modelname)`.
- 5 $\frac{2}{(s+9.997)(s+2.003)}$ is the approximate transfer function.

Aircraft Pitch Control

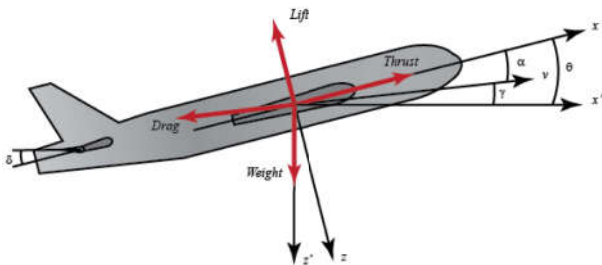


Aircraft Pitch Control



Transfer function, $G(s) = \frac{\Theta(s)}{\Delta(s)} = ??$

Aircraft Pitch Control



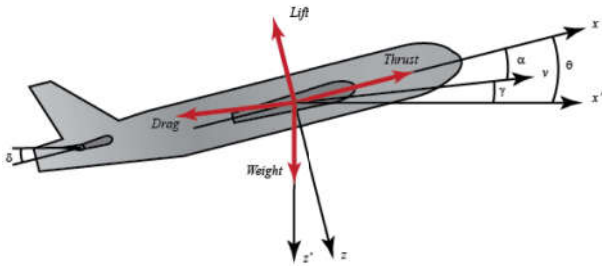
Physical Parameters

α = Angle of attack
 q = Pitch Rate
 θ = Pitch rate
 δ = Elevator deflection

Physical Parameters

ρ = Density of air
 S = Platform area of wing
 \bar{c} = Average chord length
 m = Mass of aircraft
 $\mu = \frac{\rho S \bar{c}}{4m}$
 U = Equilibrium flight speed
 C_T = Coefficient of thrust
 C_D = Coefficient of drag
 C_L = Coefficient of lift
 C_W = Coefficient of weight
 C_M = Coefficient of pitch moment
 γ = Flight path angle
 $\Omega = \frac{2U}{\bar{c}}$
 $\sigma = \frac{1}{1 + \mu C_L} = \text{constant}$
 i_{yy} = Normalized moment of inertia
 $\eta = \mu \sigma C_M = \text{constant}$

Aircraft Pitch Control



Under these assumptions, the following 3 governing equations are used by one of Boeing's commercial aircraft.

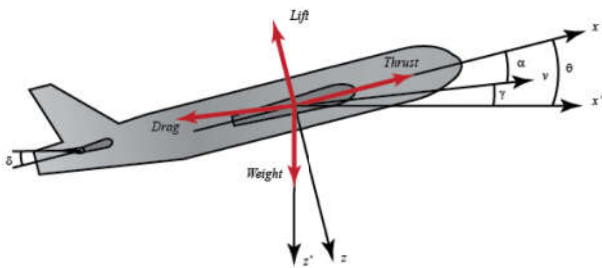
$$\frac{d\alpha}{dt} = -0.313\alpha + 56.7q + 0.232\delta$$

$$\frac{dq}{dt} = -0.0139\alpha - 4.426q + 0.0203\delta$$

$$\frac{d\theta}{dt} = 56.7q$$

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. Therefore, in order to simplify, we will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x- and y-direction and that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit!).

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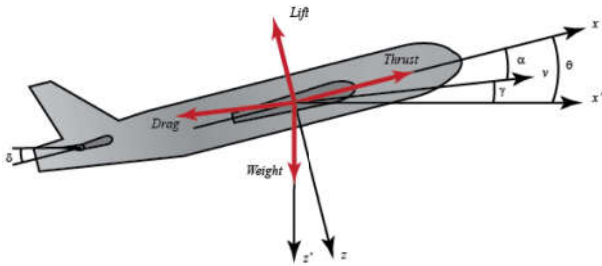
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$$\text{Ans: } G(s) = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s}$$

Aircraft Pitch Control



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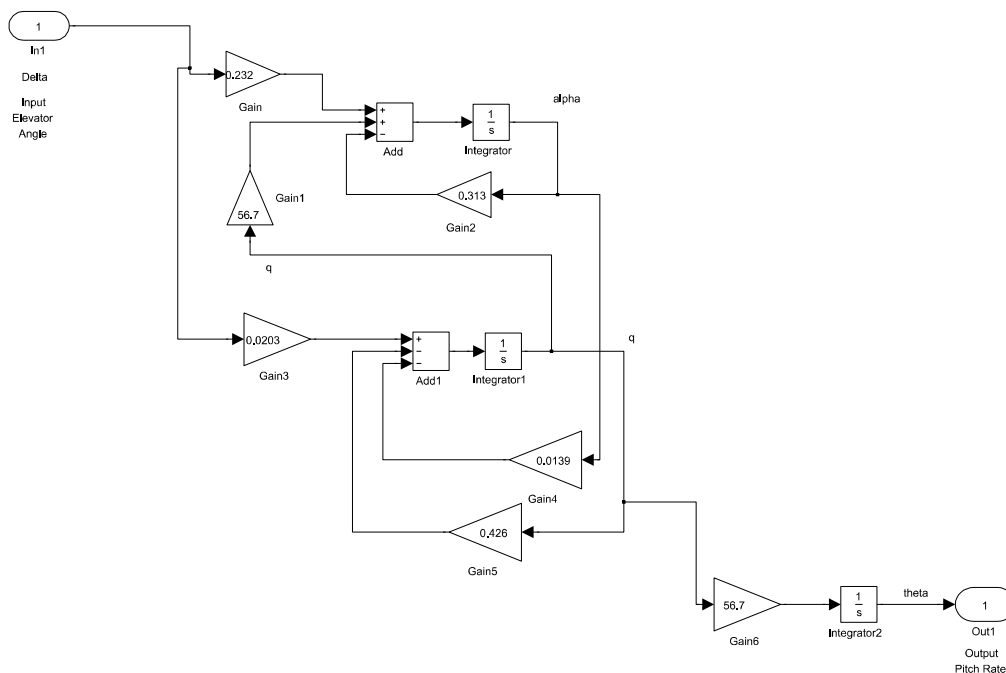
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Is the system stable?

Draw step response when elevator deflection is 0.2 rad.

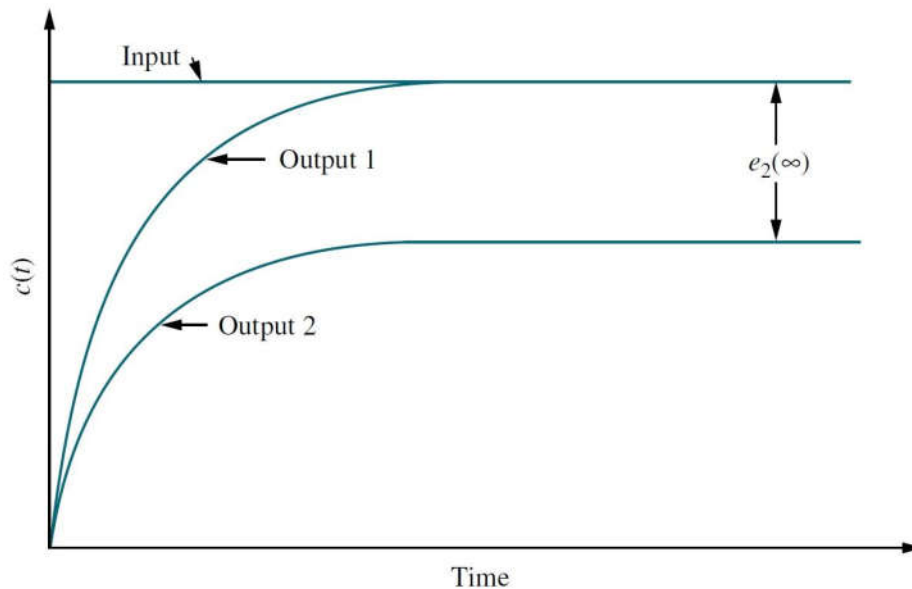
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Aircraft Pitch Control

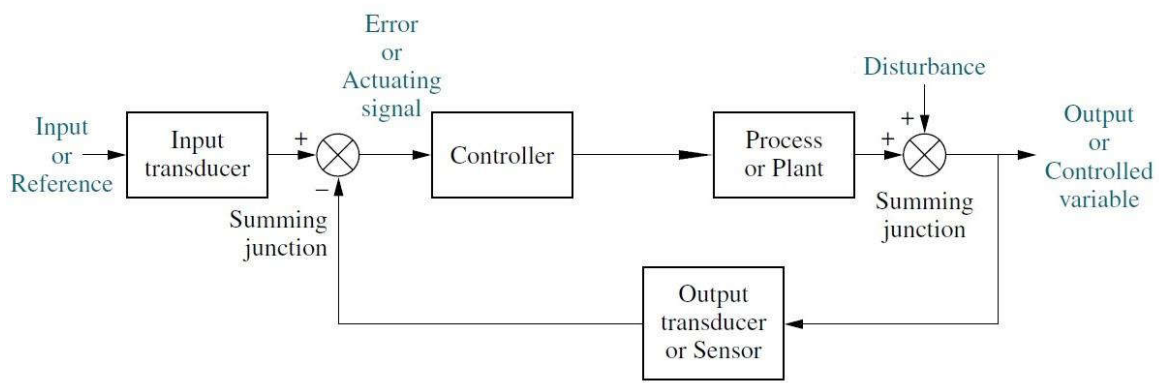


Steady State Error

The difference between the input and output for a prescribed test input as time, t approaches ∞ .



Feedback Control



closed-loop system

Feedback Control

- In feedback control , the objective is to reduce error signal to zero.

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- $e(t)$ = Error Signal
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- $y_m(t)$ = Measured value of the controlled or process variable (equivalent sensor signal)
- Although the error signal equation implies that set point is time varying but in many applications it is kept constant over a long period of time.

Different types of feedback control

Proportional Control (P)

- For Proportional Control , the objective is to reduce error signal to zero.
- $p(t) = \bar{p} + K_p e(t)$
- $p(t)$ = Controller output
- \bar{p} = Bias or steady state value
- K_p = Proportional Controller Gain

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Integral Control (I)

- For Integral Control , the rate of change of controller output is proportional to the error signal.
- $p(t) = \bar{p} + K_I \int e(t)$ i.e., $\frac{dp}{dt} = K_I e(t)$
- $p(t)$ = Controller output
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- K_I = Integral Controller Gain

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Different types of feedback control(contd.)

Derivative Control (D)

- For Derivative Control , the controller output is proportional to the rate of change of error signal.
- $p(t) = \bar{p} + K_d \frac{de(t)}{dt}$
- $p(t)$ = Controller output
- \bar{p} = Bias or steady state value
- K_d = Derivative Controller Gain

How PID affects a process/system

CL Response	Rise Time	Overshoot	Settling Time	S-S Error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small change	Decrease	Decrease	No change

- 1 A proportional controller (K_p) reduces the rise time and will reduce but never eliminate the steady-state error.
- 2 An integral control (K_i) eliminates the steady-state error for a constant or step input, but it may make the transient response slower.
- 3 A derivative control (K_d) increases the stability of the system, reducing the overshoot, and improving the transient response.
- 4 Note that these correlations may not be exactly accurate, because K_p , K_i , and K_d are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for K_i , K_p and K_d .

PID Controller Design: Step Input, Unit Feedback

```

1  %%Controller Design for Unit feedback
2  clc
3  clear all
4  disp('PID Controller Design for Unit feedback');
5  disp(' ');
6  Kp = input('Input Kp = ');
7  Ki = input('Input Ki = ');
8  Kd = input('Input Kd = ');
9
10 C=tf([Kd Kp Ki], [1 0]); % Defines the PID Controller
11
12 num = input('Input Plant TF numerator vector = ');
13 den = input('Input Plant TF denominator vector = ');
14
15 P = tf(num,den); % Defines the Process Transfer Function
16
17 step(P); % Draws step response of P
18 hold on; % Uses the same figure for next plot
19
20 T= feedback(C*P,1); % Unit feedback is fed to the controlled system
21
22 t=0:0.01:2; %Sets simulation time
23
24 step(T,t); % Draws step response of closed loop feedback system T

```

PID Controller Tuning

Problem

Manually tune a PID controller for a plant having $G(s) = \frac{1}{s^2+10s+20}$. Your goal is to achieve -

- fast rise time
- minimum overshoot
- no steady-state error

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Open Loop Response

- Put $K_P = 0$, $K_D = 0$, $K_I = 0$ in the code.
- The steady-state error is as much as 95%.
- Settling time is 1.5 sec.

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- no steady-state error

Open Loop Response

- Put $K_P = 0$, $K_D = 0$, $K_I = 0$ in the code.
- The steady-state error is as much as 95%.
- Settling time is 1.5 sec.

P-Controller

- Put $K_P = 300$ in the code.
- Steady-state error is reduced.
- Rise time is reduced.
- Overshoot increased.
- Settling time slightly reduced.

PID Controller Tuning

Problem

Manually tune a PID controller for a plant having $G(s) = \frac{1}{s^2+10s+20}$. Your goal is to achieve -

- fast rise time
- minimum overshoot
- no steady-state error

Open Loop Response

- Put $K_P = 0$, $K_D = 0$, $K_I = 0$ in the code.
- The steady-state error is as much as 95%.
- Settling time is 1.5 sec.

P-Controller

- Put $K_P = 300$ in the code.
- Steady-state error is reduced.
- Rise time is reduced.
- Overshoot increased.
- Settling time slightly reduced.

PID Controller Tuning(contd.)

PD-Controller

- Put $K_P = 300$ $K_D = 10$ in the code.
- Steady-state error is slightly reduced.
- Rise time is slightly reduced.
- Overshoot reduced.
- Settling time reduced.

PID Controller Tuning(contd.)

PD-Controller

- Put $K_P = 300$ $K_D = 10$ in the code.
- Steady-state error is slightly reduced.
- Rise time is slightly reduced.
- Overshoot reduced.
- Settling time reduced.

PI-Controller

- Put $K_P = 30$ $K_I = 70$ in the code.
- Proportional gain is reduced because integral controller alone reduces the rise time and increases the overshoot effect.
- If both of them have high values it will create a double effect.
- This controller eliminates the steady-state error.

PID Controller Tuning(contd.)

PD-Controller

- Put $K_P = 300$ $K_D = 10$ in the code.
- Steady-state error is slightly reduced.
- Rise time is slightly reduced.
- Overshoot reduced.
- Settling time reduced.

PI-Controller

- Put $K_P = 30$ $K_I = 70$ in the code.
- Proportional gain is reduced because integral controller alone reduces the rise time and increases the overshoot effect.
- If both of them have high values it will create a double effect.
- This controller eliminates the steady-state error.

PID-Controller

- Put $K_P = 350$ $K_I = 300$ $K_D = 5500$ in the code.
- Very fast rise time.
- No overshoot.
- No steady state error.